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SENSITIVITY ANALYSIS OF BOTTOM-HOLE PRESSURE CALCULATION METHODS IN FLOWING GAS WELLS

by

© GEORGE CHAN-CHUNG YEUNG

A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "SENSITIVITY ANALYSIS OF BOTTOM-HOLE PRESSURE CALCULATION METHODS IN FLOWING GAS WELLS," submitted by George Chan-Chung Yeung, in partial fulfilment of the requirements for the degree of Master of Science in Petroleum Engineering.

ABSTRACT

The effect of changes in input variables on the calculated bottom-hole pressures of flowing gas wells for six different calculation methods were investigated. The six methods studied were:

1. Average Temperature and Average Compressibility Factor Method
2. Average Density Method
3. Cullender and Smith Method
4. Poettmann Method
5. Sukkar and Cornell Method
6. Dranchuk and McFarland Method

Both the analytical and numerical approaches were used. Numerical solutions and computer programs were developed for all the six methods. However, analytical solutions were developed only for methods 1 and 2.

The input variables studied were well depth, flow rate, wellhead pressure, bottom-hole temperature, wellhead temperature, gas gravity and absolute pipe roughness. The error introduced by using linear interpolation of tabulated integral values in the Poettmann and in the Sukkar-Cornell Method was also studied.

For methods 1 and 2, the results obtained using the analytical and numerical approach are in good agreement. For all the methods studied, the results showed that the calculated bottom-hole pressure is most sensitive to variations in wellhead pressure and least sensitive to temperature variations. The error introduced by using linear

interpolation in the Poettmann Method in the pressure and temperature range studied in this work is small and thus linear interpolation can be used with confidence when using this method. However, this procedure is not recommended for use in the Sukkar-Cornell Method, for the error introduced is significant.

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CHAPTER I

INTRODUCTION

Bottom-hole pressure can be determined either by measurement with a bottom-hole pressure gauge or by calculation. Since measurement with a bottom-hole pressure gauge is costly, especially for deep wells, calculation of bottom-hole pressure is to be preferred when possible. Most of the calculation methods presently available are based on the mechanical energy balance or the general flow equation. Varying the assumptions in solving the flow equation has led to many different bottom-hole pressure calculation procedures. In all the methods, well effluent composition or gravity, wellhead pressure, flow rate, well depth and well bore temperatures are factors that must be known before calculating the bottom-hole pressure.

The most widely used and recently developed methods are:

1. Average Temperature and Average Compressibility Factor Method
2. Average Density Method
3. Cullender and Smith Method
4. Poettmann Method
5. Sukkar and Cornell Method
6. Dranchuk and McFarland Method

For Methods 1 and 2, a closed-form solution is available and thus analytical methods were developed to solve the flow equation for these two cases. For the remaining four methods, Leibniz's formula may be used to differentiate under the integral sign. If this

approach is taken, analytical equations can be developed for these four cases as well. However, the resulting equations will still have to be integrated and, consequently very little is gained by taking this approach for methods 3 to 6. Therefore numerical solutions and computer programs were developed for all the six methods to calculate the bottom-hole pressure.

All the six methods mentioned were originally developed for use up to a reduced pressure of 15 or less due to the limit of availability of the compressibility factor. This study has extended all the above methods to calculate bottom-hole pressure up to a reduced pressure of 30, using a recently developed, extended compressibility factor calculation technique. Gases containing contaminating H_2S and CO_2 were handled by applying the Wichert and Aziz [1] pseudo-critical properties modification method.

Very often the bottom-hole pressure calculated using the above methods differs from the measured value. This may be due to (a) the shortcomings of the method itself, and (b) inaccuracy in the input data required to do the calculations. The objective of this investigation is to examine the effect of (b) for all the six methods.

The input variables studied were depth, flow rate, wellhead pressure, wellhead temperature, bottom-hole temperature, gas gravity and absolute pipe roughness. The effect on the calculated bottom-hole pressure of varying each variable was studied for all the methods mentioned. The effect of absolute pipe roughness was studied for Methods 1, 2, 3 and 4. A range of pipe roughness was used as suggested by Cullender and Binkley [2], Smith and Williams [3] and Smith and

Miller [4]. The friction factor correlation used in Methods 5 and 6, which assumes a constant pipe roughness of .0006 inch, is a function of pipe diameter only. Consequently, no attempt has been made to study the effect of pipe roughness for these two methods. The effect of using linear interpolation in the Poettmann and Sukkar-Cornell Method was also examined.

CHAPTER II

LITERATURE REVIEW

A variety of methods has been developed for calculating flowing bottom-hole pressure in gas wells.

R.V. Smith [5] derived an equation for the vertical flow of gas similar to the Weymouth equation [6] for horizontal flow by assuming a constant effective compressibility factor and constant average temperature. This method is known as the Average Temperature and Average Compressibility Factor Method.

Rzasa and Katz [7] assumed a constant average pressure, temperature and compressibility factor in calculating static pressure gradient in gas wells. This method became known as the Average Density Method.

Poettmann [8] derived an expression for calculating the sandface pressure of flowing gas wells in which the variation with pressure of the compressibility factor of the gas is taken into consideration.

Cullender and Smith [9] developed a workable procedure for calculating pressures in gas wells and pipelines which makes no assumptionsregarding either temperature or compressibility. Since no assumptions are made regarding the temperature and compressibility, this method is believed to give a more accurate prediction of the bottom-hole pressure.

A method presented by Fowler [10] assumes a constant average temperature and involves integrated values of the gas law deviation

factor with pressure. It is a direct method of calculating static bottom-hole pressure. Sukkar and Cornell [11] extended Fowler's and Poettmann's methods and presented a general approach for calculating both static and flowing bottom-hole pressures for natural gas wells. However, Sukkar and Cornell only tabulated integral values for P_r from 1.0 to 12.0 and T_r from 1.5 to 1.7 and thus the method has limited applicability.

Dranchuk and McFarland [12] presented a technique for numerically integrating the differential form of the force-momentum balance in order to calculate pressure profiles in flowing gas wells for which the temperature profiles are known. The technique is similar to that presented by Young [13] except that it (a) employs a variable step-size, which reduces the number of steps required while increasing accuracy; (b) allows for the variation of both compressibility and temperature with depth in the kinetic energy term; (c) effects integration in such a manner that both production and injection cases may be treated with one equation which has no singularities.

CHAPTER III

THEORY

All of the methods studied in this work are directly or indirectly derived from the general flow Equation (1) by the introduction of certain assumptions. The derivation of each method may be outlined as follows: starting with the general flow equation which may be written as:

$$\int_1^2 Vdp + \int_1^2 \frac{udu}{g_c} + \frac{g}{g_c} \int_1^2 d\ell + \ell_w + W_s = 0 \qquad (1)$$

One may replace V by $1/\rho$, which reduces the equation to

$$\int_1^2 \frac{dP}{\rho} + \int_1^2 \frac{udu}{g_c} + \frac{g}{g_c} \int_1^2 d\ell + \ell_w + W_s = 0 \qquad (2)$$

Since there is no shaft work, $W_s = 0$; usually Δu is small, consequently methods 1 to 5 assume that the kinetic energy term can be taken as zero. By assuming $g = g_c$ numerically, Equation (2) can be rewritten as:

$$\int_1^2 \frac{dP}{\rho} + \int_1^2 d\ell + \ell_w = 0 \qquad (3)$$

The lost work term, ℓ_w, can be estimated by the following equation:

$$\ell_w = \int_1^2 \frac{fu^2}{2g_cD} dx \qquad (4)$$

The modified gas law states that

$$PV^{*} = ZnRT = \frac{w}{M} ZRT \tag{5}$$

Since density, ρ is defined as w/V^{*}, one can write

$$\rho = \frac{PM}{ZRT} \tag{6}$$

This equation can be rearranged to yield

$$\frac{P}{\rho} = \frac{ZRT}{M} \tag{7}$$

The gas gravity is defined as

$$G = \frac{M_{gas}}{M_{air}} = \frac{M_{gas}}{28.96} \tag{8}$$

By substituting this equation and the value of R, the universal gas constant, into Equation (7), one obtains

$$\frac{P}{\rho} = \frac{1545\ ZT}{28.96\ G} = 53.34\ \frac{ZT}{G} \tag{9}$$

Equations (4) and (9) can then be substituted into Equation (3) to yield

$$\int_1^2 \frac{53.34\ ZT}{G} \frac{dP}{P} + \int_1^2 d\ell + \int_1^2 \frac{fu^2}{2g_c D} d\ell = 0 \tag{10}$$

Before integrating, it should be noted that Q_o and u are interrelated through the expression

$$u = \frac{Q_o(4 \times 10^6)}{(24 \times 3600)\pi D^2} \cdot \frac{P_o}{T_o Z_o} \cdot \frac{TZ}{P} \tag{11}$$

where

Q_o = flow rate in MMSCF/D

u = velocity in ft/sec.

Assuming $Z_o = 1$ and substituting Equation (11) into Equation (10), one obtains

$$\int_1^2 \frac{53.34\ TZ}{G} \frac{dP}{P} + \int_1^2 d\ell + \int_1^2 \left[\frac{Q_o}{0.0216\ \pi D^2} \cdot \frac{P_o}{T_o} \cdot \frac{TZ}{P}\right]^2 \cdot \frac{f}{2g_c D}\, d\ell = 0 \tag{12}$$

This equation is the starting point for the derivation of methods 1 to 5. The difference between these methods arises from the manner in which the temperature and the compressibility factor are handled so as to effect integration. These differences may be summarized as follows.

1. Average Temperature and Average Compressibility Factor Method

Assumptions involved in this method are:

(a) Steady-state flow

(b) Single-phase gas flow

(c) Change in kinetic energy is small and may be neglected

(d) Constant temperature at some average value

(e) Constant compressibility at some average value

(f) Constant friction factor over the length of the conduit

Define

$$a = \frac{53.34 \langle T\rangle \langle Z\rangle}{G} \tag{13}$$

and

$$b = \frac{f}{2g_c D^5} \left[\frac{Q_o \langle T\rangle \langle Z\rangle P_o}{0.0216\ \pi T_o}\right]^2 \tag{14}$$

Substituting $T = \langle T\rangle$, $Z = \langle Z\rangle$ and Equations (13) and (14) into (12), one obtains

$$a \frac{dP}{P} + d\ell + b \frac{d\ell}{P^2} = 0 \tag{15}$$

Multiplying Equation (15) throughout by $\frac{P}{a d\ell}$ yields:

$$\frac{dP}{d\ell} + \frac{P}{a} + \frac{b}{aP} = 0 \tag{16}$$

This is an ordinary differential equation and can be solved for the following boundary conditions:

$$\ell = \ell_1 = 0 \;;\quad P = P_1$$

$$\ell = \ell_2 = L \;;\quad P = P_2$$

The solution may be written as:

$$P_2^2 = P_1^2 e^{\frac{-2L}{a}} + b\left(e^{\frac{-2L}{a}} - 1\right) \tag{17}$$

By multiplying Equation (17) throughout by $e^{\frac{2L}{a}}$ yields:

$$P_2^2 e^{\frac{2L}{a}} = P_1^2 + b\left(1 - e^{\frac{2L}{a}}\right) \tag{18}$$

By defining

$$c = \frac{2L}{a} = \frac{2GL}{53.34 <T> <Z>}$$

and substituting b and c into Equation (18), one obtains

$$e^c P_2^2 = P_1^2 + \frac{fQ_0^2 <T>^2 <Z>^2 P_0^2}{2g_c D^5 (0.0216)^2 \pi^2 T_0^2} (1 - e^c) \tag{19}$$

If one lets

P_o = 14.65 psia

T_o = 60°F = 520°R

D = d/12; where d = diameter in inches

and substitutes into Equation (19), one obtains the working equation for the Average Temperature and Average Compressibility Factor Method, which is,

$$P_1 = \left[e^c P_2^2 + \frac{25 Q_0^2 G <T> <Z> fL (e^c - 1)}{d^5 c}\right]^{0.5} \tag{20}$$

The method consists of picking an initial guess for P_1, calculating <Z>, at the average temperature and pressure, and the friction factor and calculating P_1 using equation (20). The result is compared with the initial guessed value. The procedure is repeated until a match in the value of P_1 is obtained. Data required for this method are gas gravity, flow rate, well depth, flow pipe diameter, wellhead pressure, wellhead temperature and bottom-hole temperature.

2. Average Density Method

Assumptions involved in this method are:

(a) Steady-state flow

(b) Single-phase gas flow

(c) Change in kinetic energy is small and may be neglected

(d) Gas density is constant at an average value

$$\langle\rho\rangle = (\rho_1 + \rho_2)/2$$

(e) Friction factor is constant over the length of the conduit

(f) Velocity of fluid is constant at an average value

$$u = \langle u\rangle = u \text{ at } \langle\rho\rangle$$

Recall that

$$\int_1^2 \frac{dP}{\rho} + \int_1^2 d\ell + \int_1^2 \frac{fu^2}{2g_c D} d\ell = 0 \qquad (21)$$

and

$$\rho_o = \frac{GP_o}{53.34\ T_o} \qquad (22)$$

The average density can be written as:

$$\langle\rho\rangle = \frac{\rho_1 + \rho_2}{2} = \frac{G}{(2)(53.34)^2} \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2} \right) \qquad (23)$$

Recall that

$$\langle u\rangle = \frac{4}{\pi D} \cdot \frac{Q_o \rho_o}{\langle\rho\rangle} = \frac{4 \times 10^6}{(24 \times 3600)\pi D^2} \cdot \frac{Q_o \rho_o}{\langle\rho\rangle} \qquad (24)$$

where Q_o is in MMSCF/D.

By substituting Equation (23) into Equation (24), one obtains

$$\langle u\rangle = \frac{4 \times 10^6}{(24 \times 3600)\pi D^2} \cdot \frac{Q_o G P_o}{53.34\ T_o} \cdot \frac{2 \times 53.34}{G}\left(\frac{1}{\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}}\right) \quad . . . . . (25)$$

Substituting $\langle u^2\rangle$ and $\langle\rho\rangle$ into Equation (21), one obtains

$$\int_1^2 dP = 9.374 \times 10^{-3}\ GL\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)\left[1 + \frac{Q_o^2 f}{2 \times 0.0108^2\ \pi T_o^2 g_c D^5}\left(\frac{1}{\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}}\right)^2\right] \quad (26)$$

Integration of this equation and substitution for T_o yields

$$P_1 - P_2 = 9.374 \times 10^{-3}\ GL\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right) + \frac{24.99\ Q_o^2 GLf}{d^5}\left(\frac{1}{\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}}\right) \quad (27)$$

This is the working equation for the Average Density Method. The method of solution is similar to that described in Method 1.

3. Cullender and Smith Method

Assumptions used in this method are:

(a) Steady-state flow

(b) Single-phase gas flow

(c) Change in kinetic energy is small and may be neglected

(d) Temperature gradient is known

(e) Constant friction factor over the length of the conduit

With the above assumptions, Equations (9) and (11) can be substituted into Equation (21) to yield

$$\frac{53.34}{G} \cdot \frac{TZ}{P}\,dP + d\ell + \frac{f}{2g_c D^5}\left(\frac{Q_oP_o}{0.0216\ \pi T_o}\right)^2\left(\frac{TZ}{P}\right)^2 d\ell = 0 \qquad \ldots\ldots (28)$$

This equation may be rearranged to yield

$$\frac{G}{53.34}\int_1^2 d\ell = \int_2^1 \frac{TZ/P}{\left[1 + \frac{f}{2g_c D^5}\left(\frac{Q_oP_o}{0.0216\ \pi T_o}\right)^2\left(\frac{TZ}{P}\right)^2\right]}\,dP \qquad (29)$$

Multiplication of the right hand side of this equation by $(\frac{P}{TZ})^2/(\frac{P}{TZ})^2$ produces

$$\frac{GL}{53.34} = \int_2^1 \frac{P/TZ}{\left[\left(\frac{P}{TZ}\right)^2 + \frac{f}{2g_c D^5}\left(\frac{Q_oP_o}{0.0216\ \pi T_o}\right)^2\right]}\,dP \qquad (30)$$

Substituting

$$g = g_c = 32.174$$

$$D = d/12$$

$$P_o = 14.65 \text{ psia}$$

and $$T_o = 520°R$$

one obtains

$$\frac{GL}{53.34} = \int_2^1 \frac{P/TZ}{\left[\left(\frac{P}{TZ}\right)^2 + 666.6\,\frac{fQ_o^2}{d^5}\right]}\,dP \tag{31}$$

For convenience, one may define

$$I = \frac{P/TZ}{\left[\left(\frac{P}{TZ}\right)^2 + 666.6\,\frac{fQ_o^2}{d^5}\right]}$$

in which case Equation (31) can be written as

$$\frac{GL}{53.34} = \int_2^1 I\,dP \tag{32}$$

This is the working equation for the Cullender-Smith Method. An evaluation of the integral over defined limits can be accomplished by numerical means. The method is rather tedious if a large number of increments is chosen for L. However, the authors suggest that by means of a two-step calculation and the application of Simpson's Rule, reasonable accuracy can be obtained. Thus equation (32) can be evaluated numerically using the following form:

$$\frac{2GL}{53.34} = \frac{2\Delta P}{3}\left(I_o + 4I_1 + I_2\right) \tag{33}$$

where

$$P = P_2 - P_o$$

and

P_o = wellhead pressure

P_2 = bottom-hole pressure

The method of solution consists of a trial and error evaluation of I_o, I_1 and I_2. The bottom-hole pressure can then be calculated from Equation (33).

4. Poettmann Method

This method employs the following assumptions to derive practical equations from the basic energy balance:

(a) Steady state flow

(b) Single-phase gas flow

(c) Change in kinetic energy is small and neglected

(d) Temperature is constant at some average value

(e) Friction factor is constant over the length of pipe

(f) Velocity is constant at an integrated average value

$$u = \langle u \rangle = \frac{\int_{P_2}^{P_1} u dP}{\int_{P_2}^{P_1} dP} = \frac{\int_{P_2}^{P_1} u dP}{(P_1 - P_2)} \qquad (34)$$

Equation (21) can be written in terms of <u>

$$\int_1^2 \frac{dP}{\rho} + \int_1^2 \left[1 + \frac{f\langle u\rangle^2}{2g_cD}\right] d\ell = 0 \qquad (35)$$

Combining Equations (11) and (34) and substituting T = <T>, one obtains

$$\langle u\rangle = \frac{Q_0P_0 \langle T\rangle}{0.0216\ \pi T_0D^2\Delta P} \int_2^1 \frac{Z}{P}\, dP \qquad (36)$$

where

$$\Delta P = P_1 - P_2$$

Since

$$P_r = P/P_c$$

therefore

$$dP = P_c\ dP_r \qquad (37)$$

For convenience, one may define

$$I_{p_1} = \int_{0.2}^{P_{r_1}} \frac{Z}{P_r}\, dP_r$$

where

I_{p_1} = Poettmann integral at point 1

in which case Equation (36) can be written as

$$\langle u\rangle = \frac{Q_0P_0\ \langle T\rangle}{0.0216\ \pi T_0D^2\Delta P} (I_{p_1} - I_{p_2}) \qquad (38)$$

Recall that

$$\frac{1}{\rho} = \frac{53.34 \langle T \rangle}{G} \left(\frac{Z}{P} \right)$$

therefore

$$\int_2^1 \frac{dP}{\rho} = \frac{53.34}{G} (I_{p_1} - I_{p_2}) \tag{39}$$

By substituting Equations (38) and (39) into equation (35), one can integrate the resulting equation to obtain

$$\frac{GL}{53.34 \langle T \rangle} = \frac{(I_{p_1} - I_{p_2})}{1 + \frac{f}{2g_c D^5} \left(\frac{Q_o P_o \langle T \rangle}{0.0216 \, \pi T_o \Delta P} \right)^2 (I_{p_1} - I_{p_2})^2} \tag{40}$$

By defining

$$X_s = \frac{53.34 \langle T \rangle}{G} (I_{p_1} - I_{p_2})$$

and substituting the values of T_o and P_o into Equation (40), one obtains the working equation for the Poettmann Method

$$P_1 = P_2 + \left[\frac{0.2343 \, f Q_o^2 G^2 X_s^2 L}{d^5 (X_s - L)} \right]^{0.5} \tag{41}$$

Method of solution involves trial and error with procedures as described in Method 1.

5. Sukkar and Cornell Method

This method uses the same assumptions as the Cullender and Smith Method except it makes the additional assumption that $T = \langle T \rangle = \text{constant}$.

Thus, by substituting $T = \langle T \rangle$, Equation (31) can be written as

$$\frac{GL}{53.34 \langle T \rangle} = \int_2^1 \frac{Z/P}{\left[1 + \dfrac{666.6 \; fQ_0^2 \; \langle T \rangle^2}{d^5} \left(\dfrac{Z}{P}\right)^2\right]} dP \tag{42}$$

Substituting Equation (37) into the above equation, one obtains

$$\frac{GL}{53.34 \langle T \rangle} = \int_2^1 \frac{Z/P_r}{\left[1 + \dfrac{666.6 \; fQ_0^2 \; \langle T \rangle^2}{d^5 P_c^2} \left(\dfrac{Z}{P_r}\right)^2\right]} dP_r \tag{43}$$

For convenience, one may define

$$B = \frac{666.6 \; fQ_0^2 \; \langle T \rangle^2}{d^5 P_c^2}$$

in which case Equation (43) may be written as

$$\frac{GL}{53.34 \langle T \rangle} = \int_2^1 \frac{Z/P_r}{1 + B\left(\dfrac{Z}{P_r}\right)^2} dP_r \tag{44}$$

If one defines

$$I_{sc_1} = \int_{0.2}^{P_{r_1}} \frac{Z/P_r}{1 + B\left(\frac{Z}{P_r}\right)^2} dP_r$$

and

$$I_{sc_2} = \int_{0.2}^{P_{r_2}} \frac{Z/P_r}{1 + B\left(\frac{Z}{P_r}\right)^2} dP_r$$

where

I_{sc_1} = Sukkar-Cornell integral at point 1

and

I_{sc_2} = Sukkar-Cornell integral at point 2

then Equation (44) can be written as

$$I_{sc_1} = I_{sc_2} + \frac{GL}{53.34 \langle T \rangle} \qquad (45)$$

This is the working equation for the Sukkar-Cornell Method. This is a direct calculation method which does not require trial and error. Method of solution consists of calculating I_{sc_1} from Equation (45) and obtaining the value of P_r from tabulated data that will give this integral value.

6. Dranchuk and McFarland Method

This method employs a technique to integrate the force-momentum balance equation numerically in order to calculate pressure profiles in flowing gas wells for which the temperature gradient is known. The method employs a variable step-size, which reduces the number of steps required while increasing accuracy. This method does not delete the udu term as do the other methods. It also allows for the variation of both compressibility and temperature with depth in the kinetic energy term.

The working equation for this method is

$$\frac{dP}{d\ell} = \frac{P^2}{[C_1 ZT - P^2]} \left[\frac{C_3 SP}{ZT} + \frac{C_2 ZT}{P} + \frac{C_1 Z}{P} \frac{dT}{d\ell} + \frac{C_1 T}{P} \frac{dZ}{d\ell} \right] \qquad (46)$$

where

$$C_1 = \frac{W^2 R}{g_c M}$$

$$C_2 = \frac{f W^2 R}{2 g_c D M}$$

$$C_3 = \frac{M}{R}$$

and

$$S = \frac{dz}{d\ell}$$

By choosing the usual reference conditions, field units and applying the friction factor correlation

$$f = 4f' = 0.017488\ d^{-0.224}\ ; \quad d < 4.277 \text{ ins.}$$

$$= 0.016028\ d^{-0.164}\ ; \quad d \geq 4.277 \text{ ins.}$$

as suggested by Cullender and Smith [14], the appropriate values for the three coefficients become

$$C_1 = 2.0827\,\frac{Q_o^2 G}{d^4}$$

$$C_2 = 0.21853\,\frac{Q_o^2 G}{d^{5.224}} \quad ; \quad d < 4.277 \text{ ins.}$$

$$= 0.20029\,\frac{Q_o^2 G}{d^{5.164}} \quad ; \quad d \geq 4.277 \text{ ins.}$$

and

$$C_3 = 0.018748\ G$$

where

G = gas gravity

Equation (46) along with the indicated values of the coefficients may be integrated to yield pressure profiles for both producing and injection wells.

Generally, analytical solutions are preferred if possible to furnish a general solution to a problem. However, many phenomena, such as the one encountered in this work, are non-linear and the present mathematical methods proved to be well-nigh unworkable. Or perhaps analytical solutions are available but in such a form that is inconvenient for direct interpretation numerically.

For the six methods studied in this work, closed-form solution available only for methods 1 and 2 but not for the other four methods. Therefore analytical solutions were developed for these two cases. For the Average Temperature and Average Compressibility Factor

Method, the starting point of the derivation is Equation (20). By differentiating this equation with respect to each variable studied, the following equations results. A detailed derivation is shown in Appendix C.

$$\frac{dP_1}{dL} = \frac{e^{\left(\frac{2GL}{53.34 \langle T\rangle \langle Z\rangle}\right)}\left(\frac{2\ GP_2^2}{53.34 \langle T\rangle \langle Z\rangle} + \frac{25\ Q_o^2 \langle T\rangle \langle Z\rangle\ Gf}{d^5}\right)}{2\ P_1} \qquad \ldots\ldots (47)$$

$$\frac{dP_1}{dQ_o} = \frac{25\ GQ_o fL \langle T\rangle \langle Z\rangle \left(e^{\left(\frac{2GL}{53.34 \langle T\rangle \langle Z\rangle}\right)} - 1\right)}{P_1} \qquad (48)$$

$$\frac{dP_1}{dP_2} = \frac{P_2}{P_1} e^{\left(\frac{2GL}{53.34 \langle T\rangle \langle Z\rangle}\right)} \qquad (49)$$

$$\begin{aligned}\frac{dP_1}{dT_2} = \frac{dP_2}{dT_2} = \Bigg[& e^{\frac{k_2}{\langle T\rangle \langle Z\rangle}}\left(\frac{-k_1\ k_2}{2\langle Z\rangle \langle T\rangle^2} - \frac{-k_1\ k_2}{2\langle Z\rangle \langle T\rangle^2}\left(\frac{2\langle Z\rangle}{2\langle T\rangle}\right)_{\bar{P}_r}\right. \\ & + \frac{k_3}{k_2}\langle Z\rangle \langle T\rangle^2 \left(\frac{2\langle Z\rangle}{2\langle T\rangle}\right)_{\bar{P}_r} + \frac{k_3}{k_2}\langle Z\rangle^2 \langle T\rangle \\ & - \frac{k_3}{2}\langle Z\rangle^2 \langle T\rangle - \frac{k_3}{2}\langle Z\rangle \\ & \left. - \frac{k_3}{2}\langle T\rangle \left(\frac{2\langle Z\rangle}{2\langle T\rangle}\right)_{\bar{P}_r}\right) - \frac{k_3}{2}\langle Z\rangle \langle T\rangle^2 \left(\frac{2\langle Z\rangle}{2\langle T\rangle}\right)_{\bar{P}_r} \\ & - \frac{k_3}{k_2}\langle Z\rangle^2 \langle T\rangle \Bigg] / 2P_1 \end{aligned} \qquad (50)$$

where

$$k_1 = P_2^2$$

$$k_2 = \frac{2GL}{53.34}$$

$$k_3 = \frac{25\ Q_o^2 GfL}{d^5}$$

$$\frac{dP_1}{dG} = \frac{e^{\frac{2GL}{53.34\ \langle T\rangle\ \langle Z\rangle}} \left(\frac{2LP_2}{53.34\ \langle T\rangle\ \langle Z\rangle} + \frac{25\ Q_o^2\ \langle T\rangle\ \langle Z\rangle\ fL}{d^5} \right)}{2\ P_1} \quad .\ .\ .\ .\ .(51)$$

Similarly for the Average Density Method, by differentiating P in Equation (27) with respect to each variable of interest, the following equations result. A detailed derivation is shown in Appendix C.

$$\frac{dP_1}{dL} = \frac{9.374 \times 10^{-3}\ G \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2} \right) \quad \frac{24.39\ Q_o^2 Gf}{d^5 \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2} \right)}}{1 + \frac{1}{T_1Z_1} \left(\frac{24.99\ Q_o^2 GfL}{d^5 \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2} \right)^2} - 9.374 \times 10^{-3}\ GL \right)} \qquad (52)$$

$$\frac{dP}{dQ_o} = \frac{49.98\ Q_o \left(\frac{9.374 \times 10^{-3}\ GL}{\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}} \right)}{1 - \frac{9.374 \times 10^{-3}\ GL}{T_1Z_1} + \frac{24.99\ GLfQ_o^2}{T_1Z_1 \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_1Z_1} \right)^2}} \qquad (53)$$

$$\frac{dP_1}{dP_2} = \frac{1 - \frac{1}{T_2Z_2}\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)\left[1 - \frac{\partial P_2}{Z_2}\left(\frac{\partial Z_2}{P_2}\right)_{T_2}\right]}{1 - \frac{1}{T_1Z_1}\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)} \qquad \text{.....(54)}$$

$$\frac{dP_1}{dT_1} = \frac{-\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)\left(\frac{P_1}{T_1Z_2}\right)\left[\frac{1}{T_1} + \frac{1}{Z_1}\left(\frac{\partial Z_1}{\partial T_1}\right)_{P_1}\right]}{1 - \left(\frac{1}{T_1Z_1}\right)\left(9.374 \times 10^{-3}\ GL - \frac{24.99Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)} \qquad \text{.....(55)}$$

$$\frac{dP_1}{dT_2} = \frac{-\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)\left(\frac{P_2}{T_2Z_2}\right)\left[\frac{1}{T_2} + \frac{1}{Z_2}\left(\frac{\partial Z_2}{\partial T_2}\right)_{P_2}\right]}{1 - \left(\frac{1}{T_2Z_2}\right)\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_0^2GLf}{d^5\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}\right)} \qquad \text{.....(56)}$$

$$\frac{dP_1}{dG} = \frac{9.374 \times 10^{-3} L \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2} \right) + \frac{24.99 \, Q_0^2 L f}{d^5 \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2} \right)}}{1 + \frac{G}{T_1 Z_1} \left(\frac{24.99 \, Q_0^2 L f}{d^5 \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2} \right)} - 9.374 \times 10^{-3} L \right)} \qquad (57)$$

Equations (47) to (57) can be used directly to calculate the effect of changing each variable in the calculated bottom hole pressure using either the Average Temperature and Average Compressibility Factor Method or the Average Density Method.

CHAPTER IV

CALCULATION METHODS

Computer programs were written for each of the six methods studied in this work. Correlations for pseudocritical properties, compressibility factor, viscosity and friction factor of the flowing fluid are given in the subprograms. Appendix D gives the listings of all the computer programs and subprograms used in this study.

Pseudocritical properties of the flowing fluid are calculated using either the fluid compositions or gas gravity. If fluid composition is available, Kay's mixing rule [15] is used to calculate the pseudocritical properties. The presence of sour gases are accounted for by using Wichert and Aziz [16] correction to the pseudocritical temperature and pressure. However, when the fluid composition is not available and where the gas gravity is the only known gas property, the pseudocriticals may be estimated from the correlation by Brown et al. [17]. Since this correlation is presented in graphical form, algebraic equations are written to fit the points on the graph to permit computer applications. If the mole fractions of the sour gases present are known, the pseudocriticals of sour natural gases may be estimated from the correlation by Carr et al. [18]. The estimated pseudocriticals are then corrected for the presence of CO_2 and H_2S using Wichert and Aziz [19] method.

The compressibility factor of dry sweet natural gas is calculated by using the reduced form of the B-W-R equation of state

as suggested by Dranchuk, Purvis and Robinson [20]. This method can be used for sour natural gases by using modified pseudocritical temperature and pressure, as suggested by Wichert and Aziz [21]. This correlation was tested and extended to predict compressibility factors of natural gases up to a reduced pressure of 30.

Fluid viscosity is calculated using the method suggested by Abou-Kassem [22].

Friction factor is calculated by the method suggested by Dranchuk and Abou-Kassem [23].

A numerical scheme was set up to evaluate both the Poettmann and Sukkar-Cornell integrals. The integration method employed Simpson's Rule and Newton's 3/8 Rule as is decribed in Appendix B.

For the Average Temperature and Average Compressibility Factor Method, Equations (47) to (57) can be used to study the sensitivity of each parameter. For example, by substituting the values of G, <T>, <Z>, f and L into Equation (48), the following equation results:

$$\frac{dP_1}{dQ_0} = 6990.346 \frac{Q_0}{P_1} \tag{58}$$

However, the assumption has to be made that f is constant within the range of Q_0 studied. Equation (58) can be integrated to give a general solution if the boundary conditions are known. However, for sensitivity analysis purposes, Equation (58) can be integrated within the range of limits of interest, say 10% to give,

$$P_2^2 - P_1^2 = 6990.346(Q_2^2 - Q_1^2) \tag{59}$$

where

$$Q_2 = 1.1\ Q_1$$

Equation (59) can be rearranged to give,

$$\Delta P = \frac{6990.346(Q_2 + Q_1)(Q_2 - Q_1)}{P_1 + P_2} \qquad (60)$$

where

$$\Delta P = P_2 - P_1$$

The above procedure can be applied to Equations (47) to (57) to study the sensitivity of each parameter on the calculated bottom-hole pressure. However, the assumption that changing the parameter of interest only affects the calculated P_1, has to be made in order that integration can be performed. For equations involving the gradient $\frac{\partial Z}{\partial T}$ and $\frac{\partial Z}{\partial P}$, graphical methods were used.

CHAPTER V

RESULTS AND DISCUSSIONS

Two flowing gas wells with measured bottom-hole pressures were used in this work for the purpose of study and comparison. Each variable was varied incrementally up to ±10% and the bottom-hole pressure was calculated using all the six methods. The well data are shown in Table 1. These well data and an absolute roughness of 0.0006 inch were used as the basis to calculate the deviation in the calculated bottom-hole pressure upon varying the input variable.

Tables 2 and 3 show the comparison of the bottom-hole pressure calculation methods for Wells 1 and 2 respectively. For Well 1, all the methods give bottom-hole pressures less than the measured value. Differences are within the range of 32 to 42 psia. For Well 2, all the methods give higher pressures than the measured value except for the Poettmann method which gives a bottom-hole pressure of 55 psia less than the measured value. Differences are within the range of 20-30 psia for the other methods.

The sensitivity analysis of the various methods to variations in input variables are given in Tables 4 to 15 and the results are plotted on Figures 1 to 20 (Appendix A). Table 16 gives the summary of the maximum percent deviation in the calculated bottom-hole pressure for the six methods for both wells. Table I gives the degree of sensitivity (in increasing order) and the associated maximum percent deviation in the calculated bottom-hole pressure of the various methods

TABLE 1

SUMMARY OF WELL DATA

	WELL 1	WELL 2
Well depth, ft.	4753.0	11029.0
Flow rate, MSCF/D	4292.0	17359.0
Tubing I.D., in.	2.441	2.992
Gas gravity	0.640	0.702
Wellhead temperature, °F	68.0	170.0
Bottom-hole temperature, °F	115.0	243.0
Temperature gradient, °F/ft.	0.0099	0.0066
Measured B.H.P., psia	1103.7	3690.4
Mole % CO_2	0.56	2.84
Mole % N_2	1.05	4.56
Mole % H_2S	0.00	18.33

TABLE 2

COMPARISON OF BOTTOM-HOLE PRESSURE CALCULATION METHODS FOR WELL 1

METHOD	CAL'D B.H. PRESSURE (psia)	DIFFERENCE (psia)	% DIFFERENCE*
Measured value	1103.7	-	-
Average T and Z	1071.0	32.7	2.96
Average Density	1070.7	33.0	2.99
Cullender-Smith	1070.8	32.9	2.98
Poettmann	1061.6	42.1	3.81
Sukkar-Cornell	1072.3	31.4	2.84
Dranchuk-McFarland	1067.8	35.9	3.25

* Percent difference $= \dfrac{\text{Measured P} - \text{Calculated P}}{\text{Measured P}} \times 100$

TABLE 3

COMPARISON OF BOTTOM-HOLE PRESSURE CALCULATION METHODS FOR WELL 2

METHOD	CAL'D B.H. PRESSURE (psia)	DIFFERENCE (psia)	% DIFFERENCE*
Measured value	3690.4	-	-
Average T and Z	3721.8	-31.4	-0.85
Average Density	3714.3	-23.9	-0.65
Cullender-Smith	3716.9	-26.5	-0.72
Poettmann	3635.3	55.1	1.49
Sukkar-Cornell	3720.1	-29.7	-0.80
Dranchuk-McFarland	3710.9	-20.5	-0.56

* Percent difference $= \frac{\text{Measured P - Calculated P}}{\text{Measured P}} \times 100$

TABLE 16

SUMMARY OF MAXIMUM PERCENT DEVIATION IN CALCULATED BOTTOM-HOLE PRESSURE AT ±10 PERCENT VARIATION IN INPUT VARIABLE FOR WELLS 1 AND 2

		VARIABLE											
METHOD		DEPTH		FLOW RATE		WELLHEAD P		B.H. TEMP.		WELLHEAD T		GAS GRAVITY	
		-10%	+10%	-10%	+10%	-10%	+10%	-10%	+10%	-10%	+10%	-10%	+10%
AVG. T AND Z	WELL 1	-1.70	1.71	-1.06	1.16	-8.88	9.02	0.1	-0.09	0.06	-0.06	-1.83	1.97
	WELL 2	-3.21	3.26	-1.83	1.99	-7.92	8.00	0.41	-0.36	0.28	-0.26	-3.65	4.0
AVG. DENSITY	WELL 1	-1.69	1.70	-1.06	1.15	-8.89	9.03	0.10	-0.09	0.06	-0.06	-1.82	1.95
	WELL 2	-3.16	3.20	-1.80	1.95	-7.97	8.05	0.39	-0.33	0.31	-0.26	-3.59	3.93
CULLENDER-SMITH	WELL 1	-1.70	1.70	-1.06	1.15	-8.88	9.03	0.10	-0.09	0.06	-0.06	-2.09	2.34
	WELL 2	-3.19	3.23	-1.81	1.96	-7.95	8.04	0.40	-0.35	0.29	-0.26	-3.63	3.96
POETTMANN	WELL 1	-1.61	1.63	-1.36	1.44	-8.53	8.57	0.15	-0.08	-0.05	0.06	-1.57	1.58
	WELL 2	-2.99	3.04	-2.39	2.51	-7.40	7.44	0.62	-0.32	-0.21	0.25	-2.85	2.88
SUKKAR-CORNELL	WELL 1	-1.68	1.69	-1.25	1.32	-8.75	8.82	0.05	-0.04	0.06	-0.06	-1.81	1.91
	WELL 2	-3.16	3.20	-2.12	2.23	-7.74	7.78	0.31	-0.23	0.23	-0.24	-3.49	3.71
DRANCHUK-McFARLAND	WELL 1	-1.68	1.70	-1.03	1.13	-8.93	9.08	0.68	-0.41	0.65	-0.38	-1.85	1.99
	WELL 2	-3.12	3.16	-1.73	1.88	-7.99	8.03	1.38	-0.93	1.36	-0.85	-3.50	3.15

TABLE I

DEGREE OF SENSITIVITY AND THE ASSOCIATED MAXIMUM PERCENT DEVIATION IN THE CALCULATED BOTTOM-HOLE PRESSURE OF THE VARIOUS METHODS TO DIFFERENT INPUT VARIABLES FOR WELLS 1 AND 2

(A) VARIATION IN DEPTH

	WELL 1	WELL 2
1	Poettmann (1.63%)	Poettmann (3.04%)
2	Sukkar and Cornell (1.69%)	Dranchuk and McFarland (3.16%)
3	Dranchuk and McFarland (1.70%)	Sukkar and Cornell (3.20%)
4	Average Density (1.70%)	Average Density (3.20%)
5	Cullender and Smith (1.70%)	Cullender and Smith (3.23%)
6	Average T and Z (1.71%)	Average T and Z (3.26%)

(B) VARIATION IN FLOW RATE

	WELL 1	WELL 2
1	Dranchuk and McFarland (1.13%)	Dranchuk and McFarland (1.88%)
2	Cullender and Smith (1.15%)	Average Density (1.95%)
3	Average Density (1.15%)	Cullender and Smith (1.96%)
4	Average T and Z (1.16%)	Average T and Z (1.99%)
5	Sukkar and Cornell (1.32%)	Sukkar and Cornell (2.23%)
6	Poettmann (1.36%)	Poettmann (2.51%)

TABLE I (CONTINUED)

(C) VARIATION IN WELLHEAD PRESSURE

	WELL 1	WELL 2
1	Poettmann (8.57%)	Poettmann (7.44%)
2	Sukkar and Cornell (8.82%)	Sukkar and Cornell (7.78%)
3	Average T and Z (9.02%)	Average T and Z (8.0%)
4	Cullender and Smith (9.03%)	Dranchuk and McFarland (8.03%)
5	Average Density (9.03%)	Cullender and Smith (8.04%)
6	Dranchuk and McFarland (9.08%)	Average Density (8.05%)

(D) VARIATION IN BOTTOM-HOLE TEMPERATURE

	WELL 1	WELL 2
1	Sukkar and Cornell (0.051%)	Sukkar and Cornell (0.31%)
2	Average Density (0.097%)	Average Density (0.39%)
3	Cullender and Smith (0.099%)	Cullender and Smith (0.40%)
4	Average T and Z (0.10%)	Average T and Z (0.41%)
5	Poettmann (0.145%)	Poettmann (0.62%)
6	Dranchuk and McFarland (0.68%)	Dranchuk and McFarland (1.38%)

TABLE I (CONTINUED)

(E) VARIATION IN WELLHEAD TEMPERATURE

	WELL 1	WELL 2
1	Poettmann (0.056%)	Sukkar and Cornell (0.24%)
2	Average T and Z (0.058%)	Poettmann (0.25%)
3	Cullender and Smith (0.059%)	Average T and Z (0.28%)
4	Average Density (0.061%)	Cullender and Smith (0.29%)
5	Sukkar and Cornell (0.063%)	Average Density (0.31%)
6	Dranchuk and McFarland (0.65%)	Dranchuk and McFarland (1.36%)

(F) VARIATION IN GAS GRAVITY

	WELL 1	WELL 2
1	Poettmann (1.58%)	Poettmann (2.88%)
2	Sukkar and Cornell (1.91%)	Dranchuk and McFarland (3.50%)
3	Average Density (1.95%)	Sukkar and Cornell (3.71%)
4	Average T and Z (1.97%)	Average Density (3.93%)
5	Dranchuk and McFarland (1.99%)	Cullender and Smith (3.96%)
6	Cullender and Smith (2.34%)	Average T and Z (4.0%)

TABLE II

ORDER OF SENSITIVITY OF EACH VARIABLE AND THE ASSOCIATED MAXIMUM PERCENT DEVIATION IN THE CALCULATED BOTTOM-HOLE PRESSURE FOR THE SIX METHODS

(1) AVERAGE T AND Z METHOD

	WELL 1	WELL 2
A	Wellhead Temperature (0.058%)	Wellhead Temperature (0.28%)
B	Bottom-hole Temperature (0.1%)	Bottom-hole Temperature (0.41%)
C	Flow Rate (1.16%)	Flow Rate (1.99%)
D	Depth (1.71%)	Depth (3.26%)
E	Gas Gravity (1.97%)	Gas Gravity (4.0%)
F	Wellhead Pressure (9.02%)	Wellhead Pressure (8.0%)

(2) AVERAGE DENSITY METHOD

	WELL 1	WELL 2
A	Wellhead Temperature (0.061%)	Wellhead Temperature (0.31%)
B	Bottom-hole Temperature (0.097%)	Bottom-hole Temperature (0.39%)
C	Flow Rate (1.15%)	Flow Rate (1.95%)
D	Depth (1.70%)	Depth (3.20%)
E	Gas Gravity (1.95%)	Gas Gravity (3.59%)
F	Wellhead Pressure (9.03%)	Wellhead Pressure (8.05%)

TABLE II (CONTINUED)

(3) CULLENDER AND SMITH METHOD

	WELL 1	WELL 2
A	Wellhead Temperature (0.06%)	Wellhead Temperature (0.29%)
B	Bottom-hole Temperature (0.10%)	Bottom-hole Temperature (0.40%)
C	Flow Rate (1.15%)	Flow Rate (1.96%)
D	Depth (1.70%)	Depth (3.23%)
E	Gas Gravity (2.34%)	Gas Gravity (3.96%)
F	Wellhead Pressure (9.03%)	Wellhead Pressure (8.04%)

(4) POETTMANN METHOD

	WELL 1	WELL 2
A	Wellhead Temperature (0.06%)	Wellhead Temperature (0.25%)
B	Bottom-hole Temperature (0.145%)	Bottom-hole Temperature (0.62%)
C	Flow Rate (1.44%)	Flow Rate (2.51%)
D	Gas Gravity (1.58%)	Gas Gravity (2.88%)
E	Depth (1.63%)	Depth (3.04%)
F	Wellhead Pressure (8.57%)	Wellhead Pressure (7.44%)

TABLE II (CONTINUED)

(5) SUKKAR AND CORNELL METHOD

	WELL 1	WELL 2
A	Bottom-hole Temperature (0.05%)	Wellhead Temperature (0.24%)
B	Wellhead Temperature (0.06%)	Bottom-hole Temperature (0.31%)
C	Flow Rate (1.32%)	Flow Rate (2.23%)
D	Depth (1.69%)	Depth (3.20%)
E	Gas Gravity (1.91%)	Gas Gravity (3.71%)
F	Wellhead Pressure (8.82%)	Wellhead Pressure (7.78%)

(6) DRANCHUK AND McFARLAND METHOD

	WELL 1	WELL 2
A	Wellhead Temperature (0.65%)	Wellhead Temperature (1.36%)
B	Bottom-hole Temperature (0.68%)	Bottom-hole Temperature (1.38%)
C	Flow Rate (1.13%)	Flow Rate (1.88%)
D	Depth (1.70%)	Depth (3.50%)
E	Gas Gravity (1.99%)	Gas Gravity (3.50%)
F	Wellhead Pressure (9.08%)	Wellhead Pressure (8.03%)

to different input variables for Wells 1 and 2. Table II gives the order of sensitivity (in increasing order) of each variable and the associated maximum percent deviation in the calculated bottom-hole pressure for the six methods.

It can be seen that for both wells, the calculated bottom-hole pressure is most sensitive to variation in wellhead pressure and least sensitive to temperature variations. However, Dranchuk and McFarland Method is very sensitive to temperature variations (about 10 fold for Well 1 and 5 fold for Well 2) when compared to the other methods. Among the six methods studied, Poettmann Method is least sensitive to variations in depth, wellhead pressure, wellhead temperature and gas gravity for Well 1. The same is true for Well 2 except for the variation in wellhead temperature. Methods 1, 2, 3 and 6 show the same trend for both wells regarding the order of sensitivity of the variables. They are, in the order of increasing sensitivity, wellhead temperature, bottom-hole temperature, flow rate, depth, gas gravity and wellhead pressure.

Poettmann Method and Sukkar-Cornell Method also follow the same order for Well 2. For Well 1, gas gravity is more sensitive than depth in the Poettmann Method and bottom-hole temperature is more sensitive than wellhead temperature in the Sukkar-Cornell Method. In all the six methods, the sensitivity of each variable increases, as the depth and flow rate increases, except for wellhead pressure.

Tables 17 and 18 show the effect of absolute roughness on the calculated bottom-hole pressure for the four methods studied and the results were plotted on Figures 19 and 20. The results show that the

TABLE 17

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE WITH VARIATION IN ABSOLUTE ROUGHNESS FOR WELL 1

METHOD ABSOLUTE ROUGHNESS	<T> AND <Z> METHOD	AVERAGE DENSITY METHOD	CULLENDER AND SMITH METHOD	POETTMANN METHOD
0.00055	-0.085	-0.085	-0.085	-0.107
0.00060	0.0	0.0	0.0	0.0
0.00065	0.081	0.081	0.081	0.102
0.00070	0.159	0.158	0.158	0.200
0.00075	0.233	0.232	0.232	0.292
0.00080	0.305	0.303	0.304	0.382
0.00085	0.374	0.372	0.372	0.468
0.00090	0.440	0.438	0.439	0.551
0.00095	0.505	0.502	0.503	0.631
0.00100	0.567	0.564	0.565	0.709
0.00150	1.107	1.101	1.103	1.376
0.00200	1.545	1.528	1.539	1.910

TABLE 18

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE WITH VARIATION IN ABSOLUTE ROUGHNESS FOR WELL 2

ABSOLUTE ROUGHNESS	METHOD <T> AND <Z> METHOD	AVERAGE DENSITY METHOD	CULLENDER AND SMITH METHOD	POETTMANN METHOD
0.00055	-0.153	-0.150	-0.152	-0.196
0.00060	0.0	0.0	0.0	0.0
0.00065	0.145	0.143	0.144	0.186
0.00070	0.284	0.278	0.281	0.363
0.00075	0.416	0.408	0.412	0.531
0.00080	0.542	0.532	0.537	0.692
0.00085	0.664	0.652	0.657	0.847
0.00090	0.781	0.767	0.773	1.000
0.00095	0.894	0.877	0.885	1.138
0.00010	1.003	0.984	0.993	1.276
0.00015	1.937	1.899	1.917	2.195
0.00020	2.684	2.633	2.655	2.937

Poettmann Method is more sensitive to variation in absolute roughness than the other three methods studied. Thus if the absolute roughness is in error, an error will be introduced in the calculated friction factor and thus the calculated bottom-hole pressure. This may explain the inaccuracy of this method when compared with the other methods.

In order to study the effect of using linear interpolation of tabulated integral values for the Poettmann Method and the Sukkar-Cornell Method, the bottom-hole pressure was hand calculated for Well 1 using the two methods. The integral values for both methods were estimated by linear interpolation from the table of integral values established in this work. The comparison was made for ±1.0% and ±10% variation of each variable and the results are shown in Tables 19 and 20.

Table 19 shows that the error introduced by using linear interpolation in Poettmann Method is small. The base case gives an error of only 0.006%. This is to be expected since the step-size taken in P_r and T_r is small (0.05) in evaluating the integral. However, this is not the case in the Sukkar-Cornell Method. The error introduced by using linear interpolation in the base case is 6.55% which is very significant. The reason for this is that the step-size taken in B is large (5.0). Also in the Sukkar-Cornell Method, the bottom-hole pressure is calculated from the product P_r times P_c where P_r is obtained by linear interpolation. Since P_c is of the order of 600 to 700 psia, even a small error in the calculated P_r will introduce a significant error in the calculated bottom-hole pressure. Reducing the step-size in B will require too much computer time and becomes uneconomical.

TABLE 19

EFFECT OF LINEAR INTERPOLATION - POETTMANN METHOD FOR WELL 1

	CHANGE IN VARIABLES	-10%		-1%		1%		10%	
	NO CHANGE	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.
Computer Program	1061.59								
Linear Interpolation	1061.65								
Computer Program	Depth	1044.47	-1.612	1059.87	-0.162	1063.31	0.162	1078.92	1.628
Linear Interpolation		1044.49	-1.615	1059.86	-0.168	1063.31	0.157	1078.88	1.627
Computer Program	Flow Rate	1047.16	-1.359	1060.11	-0.139	1063.08	0.140	1076.83	1.436
Linear Interpolation		1047.22	-1.359	1060.08	-0.148	1063.07	0.134	1076.84	1.431
Computer Program	Wellhead Pressure	971.02	-8.532	1052.50	-0.856	1070.67	0.855	1152.61	8.574
Linear Interpolation		971.09	-8.530	1052.50	-0.861	1070.75	0.858	1152.75	8.581
Computer Program	Bottom-hole Temperature	1063.13	0.145	1061.71	0.011	1061.47	-0.019	1060.74	-0.080
Linear Interpolation		1063.16	0.143	1061.70	0.005	1061.44	-0.011	1060.68	-0.091
Computer Program	Wellhead Temperature	1061.08	-0.048	1061.53	-0.005	1061.64	-0.005	1062.19	0.057
Linear Interpolation		1061.06	-0.055	1061.57	-0.007	1061.64	-0.001	1062.25	0.057

TABLE 20

EFFECT OF LINEAR INTERPOLATION - SUKKAR-CORNELL METHOD FOR WELL 1

	CHANGE IN VARIABLES	-10%		-1%		1%		10%	
	NO CHANGE	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.	CAL'D BHP	% DEV.
Computer Program	1072.30								
Linear Interpolation	1002.02								
Computer Program	Depth	1054.27	-1.682	1070.49	-0.169	1074.11	0.169	1090.43	1.691
Linear Interpolation		987.69	-1.430	1000.56	-0.145	1003.47	0.145	1016.58	1.453
Computer Program	Flow Rate	1058.90	-1.250	1070.94	-0.127	1073.69	0.130	1086.42	1.316
Linear Interpolation		994.47	-0.753	1001.20	-0.082	1002.85	0.083	1011.13	0.910
Computer Program	Wellhead Pressure	978.48	-8.749	1062.85	-0.882	1081.75	0.882	1166.93	8.825
Linear Interpolation		905.89	-9.594	992.27	-0.972	1011.52	0.948	1097.97	9.575
Computer Program	Bottom-hole Temperature	1072.85	0.051	1072.33	0.003	1072.24	-0.006	1071.91	-0.036
Linear Interpolation		1010.75	0.871	1002.88	0.086	1001.16	-0.086	993.59	-0.841
Computer Program	Wellhead Temperature	1072.95	0.060	1072.37	0.006	1072.24	-0.006	1071.62	-0.063
Linear Interpolation		996.44	-0.557	1001.46	-0.056	1002.57	-0.056	1007.57	-0.554

The effect of 10% change in input variable on the calculated bottom-hole pressure using the equations derived for Methods 1 and 2 are summarized in Table 21. It can be seen that the numerical results are in good agreement with that obtained by analytical methods.

The sensitivity analysis range of ±10% studied in this work is sufficiently large and should cover the likely measurement error that would normally occur in practice. The well depth is measured very accurately during well completions. The error introduced here should be minimum and should be in the order of less than 0.05% in a 10,000 ft. well. The tubing head pressure is normally measured by a dead weight gauge or Barton gauge. The accuracy of these devices should be within 2% if properly maintained and calibrated. Temperature measurement is normally measured at surface with thermocouples or temperature gauge and with bottom-hole temperature bombs at the sand face. These measuring devices are usually accurate if properly calibrated and the error introduced should be less than 2%. Flow rates are commonly measured with an orifice meter. If properly designed and located in the proper location to avoid turbulence, accuracy should be within 1 to 2%. Gas gravity is usually calculated from the gas composition obtained from gas a chromatograph. If the chromatograph is well maintained, the analysis should be very accurate and the calculated gas gravity should be within 1%.

In summary, the instrumental error introduced in measuring the various parameters is generally less than 2% if the equipment is properly maintained and calibrated. The biggest error that normally occurs in field operation is in fact human error. Therefore, if the

TABLE 21

COMPARISON OF RESULTS

NUMERICAL SOLUTION VS ANALYTICAL SOLUTION
USING DATA FROM WELL 1

	<T> AND <Z> METHOD		AVERAGE DENSITY METHOD	
VARIABLE (10% CHANGE)	NUMERICAL RESULTS	ANALYTICAL RESULTS	NUMERICAL RESULTS	ANALYTICAL RESULTS
DEPTH (L)	1.71	1.70	1.70	1.68
FLOW RATE (Q_o)	1.16	1.12	1.15	1.14
WELLHEAD P. (P_2)	9.02	8.87	9.03	8.61
B.H. TEMP. (T_1)	-0.092	-0.081	-0.086	-0.083
W.H. TEMP. (T_2)	-0.055	-0.048	-0.057	-0.053
GRAVITY (G)	1.97	1.70	1.95	1.71

calculated bottom-hole pressure calculated by any one of the six methods is significantly different from that of the measured value, the field engineer should double check the various input parameters, especially the wellhead pressure and flow rate, which are most susceptible to human error, to ensure that the data is correct.

CHAPTER VI

CONCLUSIONS

The following conclusions are made as a result of this study:

1. Calculated bottom-hole pressures by all the methods studied are most sensitive to variations in wellhead pressure and least sensitive to variations in temperature.
2. The percent change in calculated bottom-hole pressure shows the same trend for all the methods except for variations in bottom-hole and wellhead temperature in the Poettmann Method.
3. The Poettmann Method is more sensitive to variations in absolute roughness than the other three methods studied.
4. At high flow rates, the kinetic energy term may be significant, thus the Dranchuk and McFarland Method should give the best results.
5. Linear interpolation can be used with confidence in the Poettmann Method.
6. Linear interpolation should not be used in the Sukkar and Cornell Method.

7. For the Average Temperature and Average Compressibility Factor Method and the Average Density Method, the numerical results obtained are in good agreement with the analytical results using the analytical solutions developed in this work.

CHAPTER VII

RECOMMENDATIONS

The following recommendations are made:

1. Collect flow data for more wells with different depths and flow rates and perform bottom-hole pressure calculations with the six methods studied. This will enable one to make a better comparison of the different methods and the limits of their applicabilities.
2. Investigate possible instrumental error involved in various measuring devices that are used to measure the input variables required to do bottom-hole pressure calculations.

NOMENCLATURE

d = inside pipe diameter, inches.

D = inside pipe diameter, feet.

f = Moody friction factor = $4\ f'$, dimensionless.

f' = Fanning friction factor, dimensionless.

g = local gravitational acceleration, 32.174 ft/sec^2.

g_c = conversion factor, 32.174 $\text{lb}_m/\text{ft}/\text{lb}_f\text{sec}^2$.

G = gas gravity (air = 1).

ℓ = length variable, feet.

ℓ_w = lost work, $\text{ft.lb}_f/\text{lb}_m$.

L = length of flow string, feet.

M = molecular weight, $\text{lb}_m/\text{lb-mole}$.

P = pressure variable, psia.

P_c = pseudocritical pressure, psia.

P_o = pressure at reference conditions, 14.65 psia.

P_r = pseudoreduced pressure, dimensionless.

Q_o = flow rate, MMSCF/Day.

R = Universal gas constant, 1545 $\text{ft.lb}_f/\text{lb-mole}^\circ\text{R}$

T = temperature, °R

T_c = pseudocritical temperature, °R

T_o = temperature at reference conditions, 520°R

$\langle T\rangle$ = average temperature, °R

u = velocity, ft/sec.

$\langle u\rangle$ = average velocity, ft/sec.

V = specific volume, ft^3/lb_m.

V^* = volume, ft^3.

W = mass velocity, lb_m/ft^2-sec.

w = mass, lb_m

W_s = shaft work, ft-lb_f/lb_m.

z = elevation

Z = compressibility factor, dimensionless.

Z_o = compressibility factor at standard conditions, dimensionless.

$\langle Z \rangle$ = average compressibility factor, dimensionless.

Greek

ρ = gas density, lb_m/ft^3.

ρ_o = gas density at standard conditions, lb_m/ft^3.

$\langle \rho \rangle$ = average gas density, lb_m/ft^3.

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APPENDICES

APPENDIX A

TABULATED RESULTS AND GRAPHS

TABLE 4

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE AVERAGE TEMPERATURE AND Z METHOD - WELL 1

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-1.70	-1.06	-8.88	0.100	0.058	-1.83
- 9.0	-1.53	-0.96	-8.00	0.089	0.052	-1.65
- 8.0	-1.36	-0.86	-7.12	0.079	0.046	-1.47
- 7.0	-1.19	-0.75	-6.23	0.069	0.040	-1.29
- 6.0	-1.02	-0.65	-5.35	0.059	0.034	-1.11
- 5.0	-0.85	-0.54	-4.46	0.049	0.029	-0.93
- 4.0	-0.68	-0.44	-3.57	0.039	0.023	-0.74
- 3.0	-0.51	-0.33	-2.68	0.029	0.017	-0.56
- 2.0	-0.34	-0.22	-1.79	0.019	0.011	-0.37
- 1.0	-0.17	-0.11	-0.90	0.010	0.006	-0.19
1.0	0.17	0.11	0.90	-0.010	-0.006	0.19
2.0	0.34	0.22	1.79	-0.019	-0.011	0.38
3.0	0.51	0.34	2.69	-0.028	-0.017	0.57
4.0	0.68	0.45	3.59	-0.038	-0.022	0.77
5.0	0.85	0.57	4.50	-0.047	-0.028	0.96
6.0	1.03	0.68	5.40	-0.056	-0.033	1.16
7.0	1.20	0.80	6.30	-0.065	-0.039	1.36
8.0	1.37	0.92	7.21	-0.074	-0.044	1.56
9.0	1.54	1.04	8.12	-0.083	-0.050	1.76
10.0	1.71	1.16	9.02	-0.092	-0.055	1.97

TABLE 5

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE AVERAGE DENSITY METHOD - WELL 1

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-1.69	-1.06	-8.89	0.097	0.061	-1.82
- 9.0	-1.52	-0.95	-8.01	0.087	0.055	-1.64
- 8.0	-1.35	-0.85	-7.13	0.077	0.049	-1.46
- 7.0	-1.18	-0.75	-6.24	0.067	0.042	-1.29
- 6.0	-1.01	-0.64	-5.36	0.057	0.036	-1.10
- 5.0	-0.85	-0.54	-4.47	0.047	0.030	-0.92
- 4.0	-0.68	-0.43	-3.57	0.038	0.024	-0.74
- 3.0	-0.51	-0.33	-2.68	0.028	0.018	-0.56
- 2.0	-0.34	-0.22	-1.79	0.019	0.012	-0.37
- 1.0	-0.17	-0.11	-0.90	0.009	0.006	-0.19
1.0	0.17	0.11	0.90	-0.009	-0.006	0.19
2.0	0.34	0.22	1.80	-0.018	-0.012	0.38
3.0	0.51	0.34	2.70	-0.027	-0.017	0.57
4.0	0.68	0.45	3.60	-0.036	-0.023	0.76
5.0	0.85	0.56	4.50	-0.044	-0.029	0.96
6.0	1.02	0.68	5.40	-0.053	-0.035	1.15
7.0	1.19	0.80	6.31	-0.062	-0.040	1.35
8.0	1.3	0.91	7.21	-0.070	-0.046	1.55
9.0	1.53	1.03	8.12	-0.078	-0.051	1.75
10.0	1.70	1.15	9.03	-0.086	-0.057	1.95

TABLE 6

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE
CULLENDER AND SMITH METHOD - WELL 1

	INPUT VARIABLE					
PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-1.70	-1.06	-8.88	0.099	0.059	-2.09
- 9.0	-1.53	-0.96	-8.00	0.089	0.053	-1.90
- 8.0	-1.36	-0.85	-7.12	0.079	0.047	-1.69
- 7.0	-1.19	-0.75	-6.24	0.069	0.041	-1.48
- 6.0	-1.02	-0.65	-5.35	0.059	0.035	-1.28
- 5.0	-0.85	-0.54	-4.46	0.049	0.029	-1.07
- 4.0	-0.69	-0.43	-3.57	0.039	0.023	-0.86
- 3.0	-0.51	-0.33	-2.68	0.029	0.017	-0.65
- 2.0	-0.35	-0.22	-1.79	0.019	0.012	-0.43
- 1.0	-0.17	-0.11	-0.90	0.010	0.006	-0.22
1.0	0.16	0.11	0.90	-0.009	-0.006	0.23
2.0	0.34	0.22	1.80	-0.019	-0.011	0.44
3.0	0.50	0.34	2.70	-0.028	-0.017	0.67
4.0	0.68	0.45	3.60	-0.037	-0.023	0.91
5.0	0.84	0.56	4.50	-0.046	-0.028	1.13
6.0	1.02	0.68	5.40	-0.055	-0.034	1.37
7.0	1.19	0.80	6.31	-0.064	-0.039	1.60
8.0	1.36	0.91	7.21	-0.073	-0.045	1.85
9.0	1.53	1.03	8.12	-0.082	-0.050	2.10
10.0	1.70	1.15	9.03	-0.090	-0.055	2.34

TABLE 7

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE
POETTMANN METHOD - WELL 1

PERCENT VARIATION	INPUT VARIABLE	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0		-1.61	-1.36	-8.53	0.145	-0.047	-1.57
- 9.0		-1.45	-1.23	-7.68	0.121	-0.043	-1.41
- 8.0		-1.29	-1.09	-6.83	0.110	-0.038	-1.26
- 7.0		-1.13	-0.96	-5.98	0.093	-0.034	-1.10
- 6.0		-0.97	-0.82	-5.13	0.078	-0.029	-0.94
- 5.0		-0.81	-0.69	-4.27	0.063	-0.025	-0.79
- 4.0		-0.65	-0.55	-3.42	0.049	-0.020	-0.63
- 3.0		-0.49	-0.42	-2.56	0.036	-0.015	-0.47
- 2.0		-0.32	-0.28	-1.71	0.023	-0.010	-0.32
- 1.0		-0.16	-0.14	-0.86	0.011	-0.005	-0.16
1.0		0.16	0.14	0.86	-0.011	0.005	0.16
2.0		0.32	0.28	1.71	-0.020	0.010	0.31
3.0		0.49	0.42	2.57	-0.030	0.016	0.47
4.0		0.65	0.57	3.43	-0.039	0.021	0.63
5.0		0.81	0.71	4.28	-0.047	0.027	0.79
6.0		0.97	0.85	5.14	-0.055	0.033	0.95
7.0		1.14	1.00	6.00	-0.062	0.038	1.10
8.0		1.30	1.14	6.86	-0.068	0.044	1.26
9.0		1.46	1.29	7.71	-0.074	0.050	1.42
10.0		1.63	1.44	8.57	-0.080	0.056	1.58

TABLE 8

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE
SUKKAR AND CORNELL METHOD - WELL 1

	INPUT VARIABLE					
PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-1.68	-1.25	-8.75	0.051	0.060	-1.81
- 9.0	-1.52	-1.13	-7.88	0.045	0.054	-1.63
- 8.0	-1.35	-1.01	-7.01	0.039	0.048	-1.45
- 7.0	-1.18	-0.88	-6.13	0.033	0.042	-1.27
- 6.0	-1.01	-0.76	-5.26	0.030	0.036	-1.10
- 5.0	-0.85	-0.63	-4.39	0.024	0.030	-0.91
- 4.0	-0.68	-0.51	-3.51	0.018	0.024	-0.73
- 3.0	-0.51	-0.38	-2.63	0.012	0.018	-0.55
- 2.0	-0.34	-0.26	-1.76	0.009	0.012	-0.37
- 1.0	-0.17	-0.13	-0.88	0.003	0.006	-0.18
1.0	0.17	0.13	0.88	-0.006	-0.006	0.19
2.0	0.34	0.26	1.76	-0.009	-0.012	0.37
3.0	0.51	0.39	2.64	-0.012	-0.018	0.57
4.0	0.68	0.52	3.52	-0.018	-0.024	0.76
5.0	0.85	0.65	4.40	-0.021	-0.030	0.94
6.0	1.01	0.78	5.28	-0.024	-0.039	1.14
7.0	1.18	0.91	6.17	-0.027	-0.045	1.33
8.0	1.35	1.05	7.06	-0.030	-0.051	1.52
9.0	1.52	1.18	7.94	-0.033	-0.057	1.72
10.0	1.69	1.32	8.82	-0.036	-0.063	1.91

TABLE 9

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE
DRANCHUK AND McFARLAND METHOD - WELL 1

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-1.68	-1.03	-8.93	0.68	0.65	-1.85
- 9.0	-1.52	-0.93	-8.05	0.60	0.57	-1.67
- 8.0	-1.35	-0.83	-7.16	0.51	0.49	-1.49
- 7.0	-1.18	-0.73	-6.27	0.44	0.41	-1.30
- 6.0	-1.01	-0.63	-5.38	0.36	0.34	-1.12
- 5.0	-0.84	-0.53	-4.49	0.30	0.28	-0.94
- 4.0	-0.68	-0.42	-3.59	0.23	0.22	-0.75
- 3.0	-0.51	-0.32	-2.70	0.17	0.16	-0.57
- 2.0	-0.34	-0.21	-1.80	0.11	0.10	-0.38
- 1.0	-0.17	-0.11	-0.90	0.05	0.05	-0.19
1.0	0.17	0.11	0.90	-0.05	-0.05	0.19
2.0	0.34	0.22	1.81	-0.10	-0.09	0.38
3.0	0.51	0.33	2.71	-0.15	-0.13	0.58
4.0	0.68	0.44	3.62	-0.19	-0.17	0.78
5.0	0.85	0.55	4.52	-0.23	-0.21	0.97
6.0	1.02	0.67	5.43	-0.27	-0.25	1.17
7.0	1.19	0.78	6.34	-0.31	-0.28	1.37
8.0	1.36	0.89	7.25	-0.35	-0.32	1.58
9.0	1.53	1.01	8.17	-0.38	-0.35	1.78
10.0	1.70	1.13	9.08	-0.41	-0.38	1.89

TABLE 10

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE AVERAGE TEMPERATURE AND Z METHOD - WELL 2

	INPUT VARIABLE					
PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-3.21	-1.83	-7.92	0.41	0.28	-3.65
- 9.0	-2.89	-1.66	-7.14	0.37	0.25	-3.30
- 8.0	-2.58	-1.48	-6.35	0.32	0.22	-2.94
- 7.0	-2.26	-1.30	-5.56	0.28	0.19	-2.59
- 6.0	-1.93	-1.12	-4.77	0.24	0.17	-2.23
- 5.0	-1.61	-0.94	-3.98	0.20	0.14	-1.86
- 4.0	-1.29	-0.75	-3.18	0.16	0.11	-1.49
- 3.0	-0.97	-0.57	-2.39	0.12	0.08	-1.13
- 2.0	-0.65	-0.38	-1.59	0.08	0.05	-0.76
- 1.0	-0.32	-0.19	-0.80	0.04	0.03	-0.38
1.0	0.32	0.19	0.80	-0.04	-0.03	0.38
2.0	0.65	0.38	1.60	-0.08	-0.05	0.77
3.0	0.97	0.58	2.39	-0.11	-0.08	1.16
4.0	1.30	0.78	3.19	-0.15	-0.11	1.55
5.0	1.63	0.97	3.99	-0.19	-0.13	1.95
6.0	1.95	1.17	4.80	-0.22	-0.16	2.35
7.0	2.28	1.37	5.60	-0.26	-0.18	2.76
8.0	2.61	1.58	6.40	-0.29	-0.21	3.17
9.0	2.93	1.78	7.20	-0.33	-0.23	3.58
10.0	3.26	1.99	8.00	-0.36	-0.26	4.00

TABLE 11

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE AVERAGE DENSITY METHOD - WELL 2

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-3.16	-1.80	-7.97	0.39	0.31	-3.59
- 9.0	-2.85	-1.62	-7.18	0.35	0.28	-3.25
- 8.0	-2.53	-1.45	-6.38	0.31	0.24	-2.90
- 7.0	-2.22	-1.27	-5.59	0.27	0.21	-2.55
- 6.0	-1.90	-1.10	-4.79	0.23	0.18	-2.19
- 5.0	-1.59	-0.92	-4.00	0.19	0.15	-1.83
- 4.0	-1.27	-0.74	-3.20	0.15	0.12	-1.47
- 3.0	-0.92	-0.56	-2.40	0.11	0.09	-1.11
- 2.0	-0.64	-0.37	-1.60	0.07	0.06	-0.74
- 1.0	-0.32	-0.19	-0.80	0.04	0.03	-0.37
1.0	0.32	0.19	0.80	-0.04	-0.03	0.38
2.0	0.64	0.38	1.61	-0.07	-0.06	0.76
3.0	0.96	0.57	2.41	-0.10	-0.08	1.14
4.0	1.28	0.76	3.21	-0.14	-0.11	1.53
5.0	1.60	0.96	4.02	-0.17	-0.14	1.92
6.0	1.92	1.15	4.82	-0.20	-0.16	2.31
7.0	2.24	1.35	5.63	-0.24	-0.19	2.71
8.0	2.56	1.55	6.44	-0.27	-0.21	3.11
9.0	2.88	1.75	7.24	-0.30	-0.24	3.52
10.0	3.20	1.95	8.05	-0.33	-0.26	3.93

TABLE 12

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE CULLENDER AND SMITH METHOD - WELL 2

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-3.19	-1.81	-7.95	0.40	0.29	-3.63
- 9.0	-2.86	-1.63	-7.16	0.36	0.26	-3.27
- 8.0	-2.55	-1.46	-6.37	0.32	0.23	-2.93
- 7.0	-2.23	-1.28	-5.58	0.28	0.20	-2.58
- 6.0	-1.92	-1.10	-4.79	0.23	0.17	-2.21
- 5.0	-1.59	-0.92	-3.99	0.19	0.14	-1.86
- 4.0	-1.28	-0.74	-3.19	0.15	0.11	-1.49
- 3.0	-0.95	-0.56	-2.40	0.11	0.08	-1.13
- 2.0	-0.64	-0.37	-1.60	0.08	0.06	-0.75
- 1.0	-0.31	-0.19	-0.80	0.04	0.03	-0.38
1.0	0.33	0.19	0.80	-0.04	-0.03	0.37
2.0	0.64	0.38	1.60	-0.07	-0.05	0.77
3.0	0.97	0.57	2.40	-0.11	-0.08	1.15
4.0	1.29	0.77	3.21	-0.15	-0.11	1.53
5.0	1.62	0.96	4.01	-0.18	-0.13	1.93
6.0	1.93	1.16	4.81	-0.22	-0.16	2.32
7.0	2.26	1.36	5.62	-0.25	-0.19	2.74
8.0	2.58	1.56	6.42	-0.28	-0.21	3.13
9.0	2.91	1.76	7.23	-0.32	-0.24	3.55
10.0	3.23	1.96	8.04	-0.35	-0.26	3.96

TABLE 13

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE
POETTMANN METHOD - WELL 2

PERCENT VARIATION	INPUT VARIABLE					
	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-2.99	-2.39	-7.40	0.62	-0.21	-2.85
- 9.0	-2.69	-2.15	-6.67	0.54	-0.19	-2.55
- 8.0	-2.40	-1.92	-5.93	0.46	-0.17	-2.28
- 7.0	-2.10	-1.68	-5.19	0.39	-0.15	-2.00
- 6.0	-1.80	-1.45	-4.45	0.33	-0.13	-1.71
- 5.0	-1.50	-1.21	-3.71	0.26	-0.11	-1.43
- 4.0	-1.20	-0.97	-2.97	0.20	-0.09	-1.14
- 3.0	-0.90	-0.73	-2.23	0.15	-0.07	-0.86
- 2.0	-0.60	-0.49	-1.48	0.10	-0.05	-0.57
- 1.0	-0.30	-0.24	-0.74	0.05	-0.02	-0.29
1.0	0.30	0.25	0.74	-0.04	0.02	0.29
2.0	0.60	0.49	1.49	-0.08	0.05	0.57
3.0	0.91	0.74	2.23	-0.12	0.07	0.86
4.0	1.21	0.99	2.97	-0.16	0.10	1.15
5.0	1.51	1.24	3.72	-0.19	0.12	1.44
6.0	1.82	1.49	4.46	-0.22	0.15	1.72
7.0	2.12	1.74	5.20	-0.25	0.17	2.01
8.0	2.43	2.00	5.95	-0.27	0.20	2.30
9.0	2.73	2.25	6.69	-0.30	0.22	2.59
10.0	3.04	2.51	7.44	-0.32	0.25	2.88

TABLE 14

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE SUKKAR AND CORNELL METHOD - WELL 2

INPUT VARIABLE PERCENT VARIATION	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-3.16	-2.12	-7.74	0.31	0.23	-3.49
- 9.0	-2.84	-1.91	-6.97	0.28	0.21	-3.15
- 8.0	-2.53	-1.71	-6.20	0.24	0.19	-2.81
- 7.0	-2.21	-1.50	-5.42	0.21	0.16	-2.47
- 6.0	-1.90	-1.29	-4.65	0.17	0.14	-2.12
- 5.0	-1.58	-1.08	-3.88	0.14	0.12	-1.77
- 4.0	-1.27	-0.86	-3.10	0.11	0.09	-1.42
- 3.0	-0.95	-0.65	-2.33	0.08	0.07	-1.07
- 2.0	-0.64	-0.43	-1.55	0.05	0.05	-0.71
- 1.0	-0.32	-0.22	-0.78	0.03	0.02	-0.36
1.0	0.32	0.22	0.78	-0.03	-0.02	0.36
2.0	0.64	0.44	1.55	-0.05	-0.05	0.73
3.0	0.95	0.66	2.33	-0.08	-0.07	1.09
4.0	1.27	0.88	3.11	-0.10	-0.10	1.46
5.0	1.59	1.10	3.88	-0.12	-0.12	1.83
6.0	1.91	1.32	4.66	-0.15	-0.15	2.20
7.0	2.23	1.55	5.44	-0.17	-0.17	2.58
8.0	2.55	1.78	6.22	-0.19	-0.19	2.95
9.0	2.88	2.00	7.00	-0.21	-0.22	3.33
10.0	3.20	2.23	7.78	-0.23	-0.24	3.71

TABLE 15

PERCENT CHANGE IN CALCULATED BOTTOM-HOLE PRESSURE DRANCHUK AND McFARLAND METHOD - WELL 2

PERCENT VARIATION / INPUT VARIABLE	DEPTH	FLOW RATE	WELLHEAD PRESSURE	BOTTOM-HOLE TEMP.	WELLHEAD TEMP.	GAS GRAVITY
-10.0	-3.12	-1.73	-7.99	1.38	1.36	-3.50
- 9.0	-2.81	-1.57	-7.19	1.22	1.19	-3.16
- 8.0	-2.50	-1.40	-6.40	1.06	1.03	-2.81
- 7.0	-2.19	-1.23	-5.60	0.91	0.88	-2.47
- 6.0	-1.87	-1.06	-4.80	0.76	0.74	-2.13
- 5.0	-1.56	-0.89	-4.00	0.62	0.60	-1.78
- 4.0	-1.25	-0.71	-3.20	0.49	0.47	-1.43
- 3.0	-0.94	-0.54	-2.40	0.36	0.34	-1.07
- 2.0	-0.63	-0.36	-1.60	0.23	0.22	-0.72
- 1.0	-0.31	-0.18	-0.80	0.12	0.11	-0.36
1.0	0.31	0.18	0.80	-0.11	-0.10	0.36
2.0	0.63	0.36	1.60	-0.22	-0.20	0.73
3.0	0.94	0.55	2.41	-0.32	-0.30	1.10
4.0	1.26	0.74	3.21	-0.42	-0.39	1.47
5.0	1.57	0.92	4.01	-0.51	-0.47	1.84
6.0	1.89	1.11	4.82	-0.60	-0.56	2.22
7.0	2.21	1.30	5.62	-0.69	-0.64	2.60
8.0	2.52	1.50	6.43	-0.76	-0.71	2.98
9.0	2.84	1.69	7.23	-0.86	-0.78	3.36
10.0	3.16	1.88	8.03	-0.93	-0.85	3.15

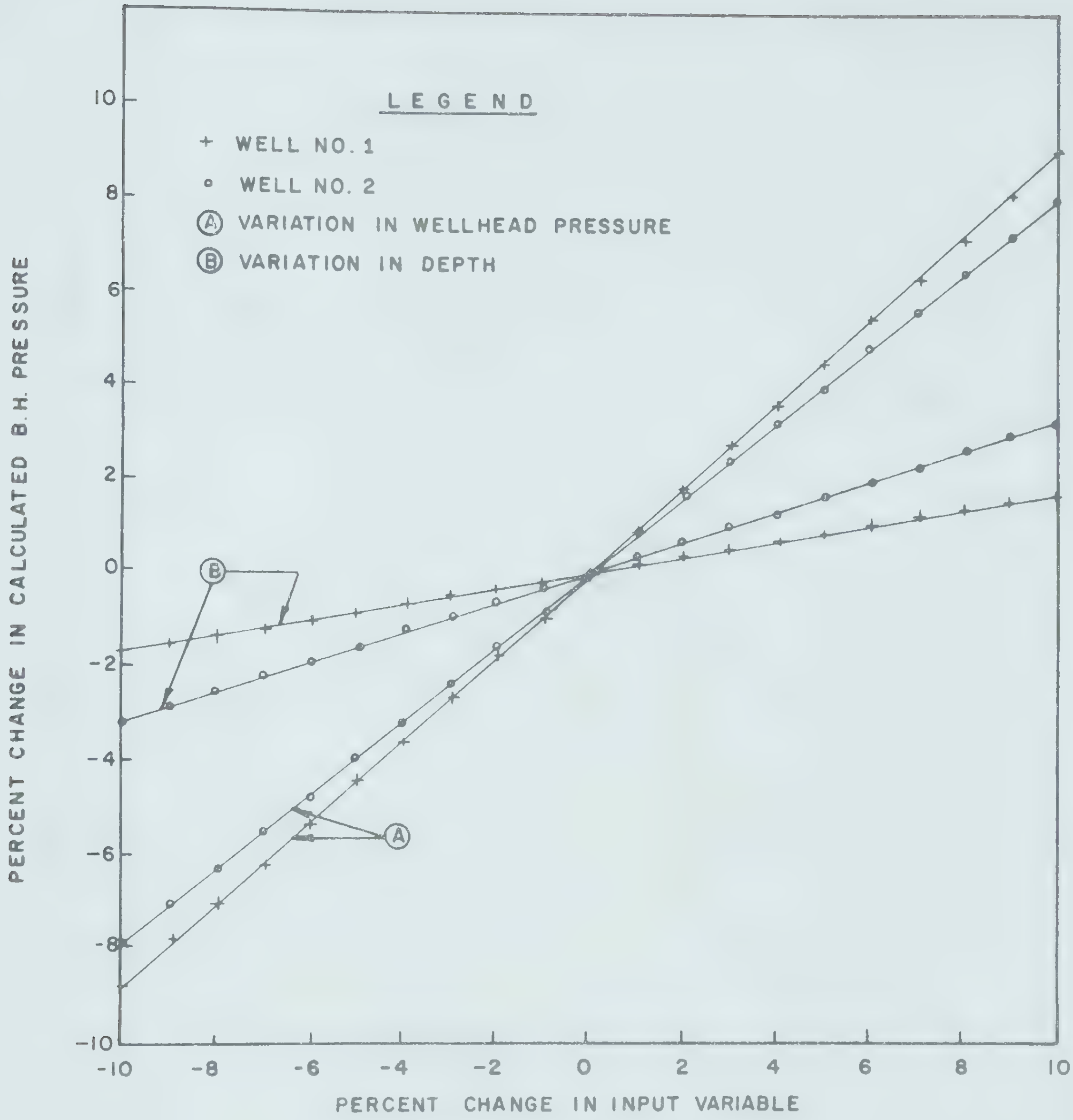

FIGURE 1 SENSITIVITY ANALYSIS OF AVERAGE TEMP. AND AVERAGE COMPRESSIBILITY FACTOR METHOD-VARIATION IN DEPTH AND WELLHEAD PRESSURE

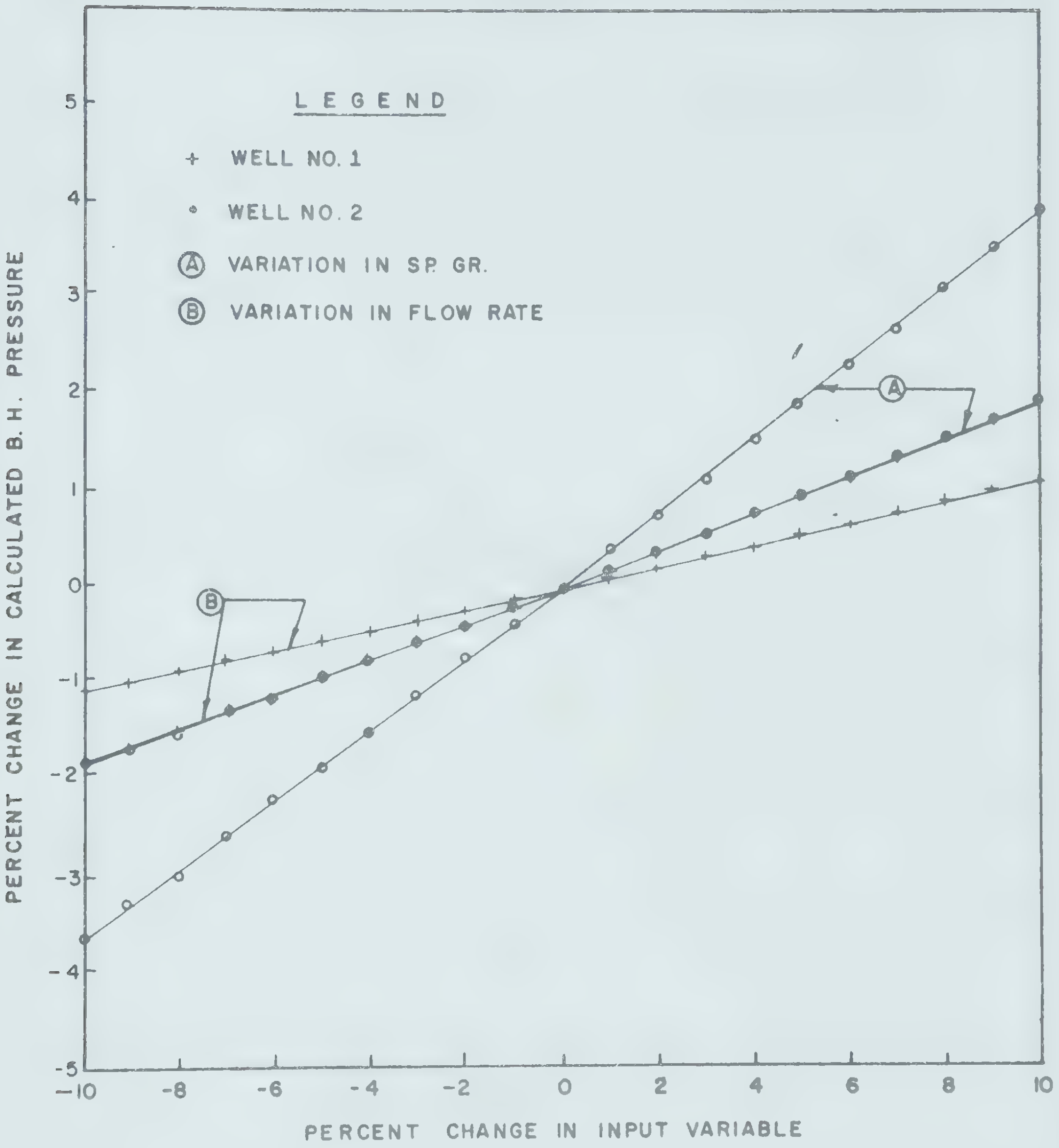

FIGURE 2 SENSITIVITY ANALYSIS OF AVERAGE TEMPERATURE AND AVERAGE COMPRESSIBILITY FACTOR METHOD - VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

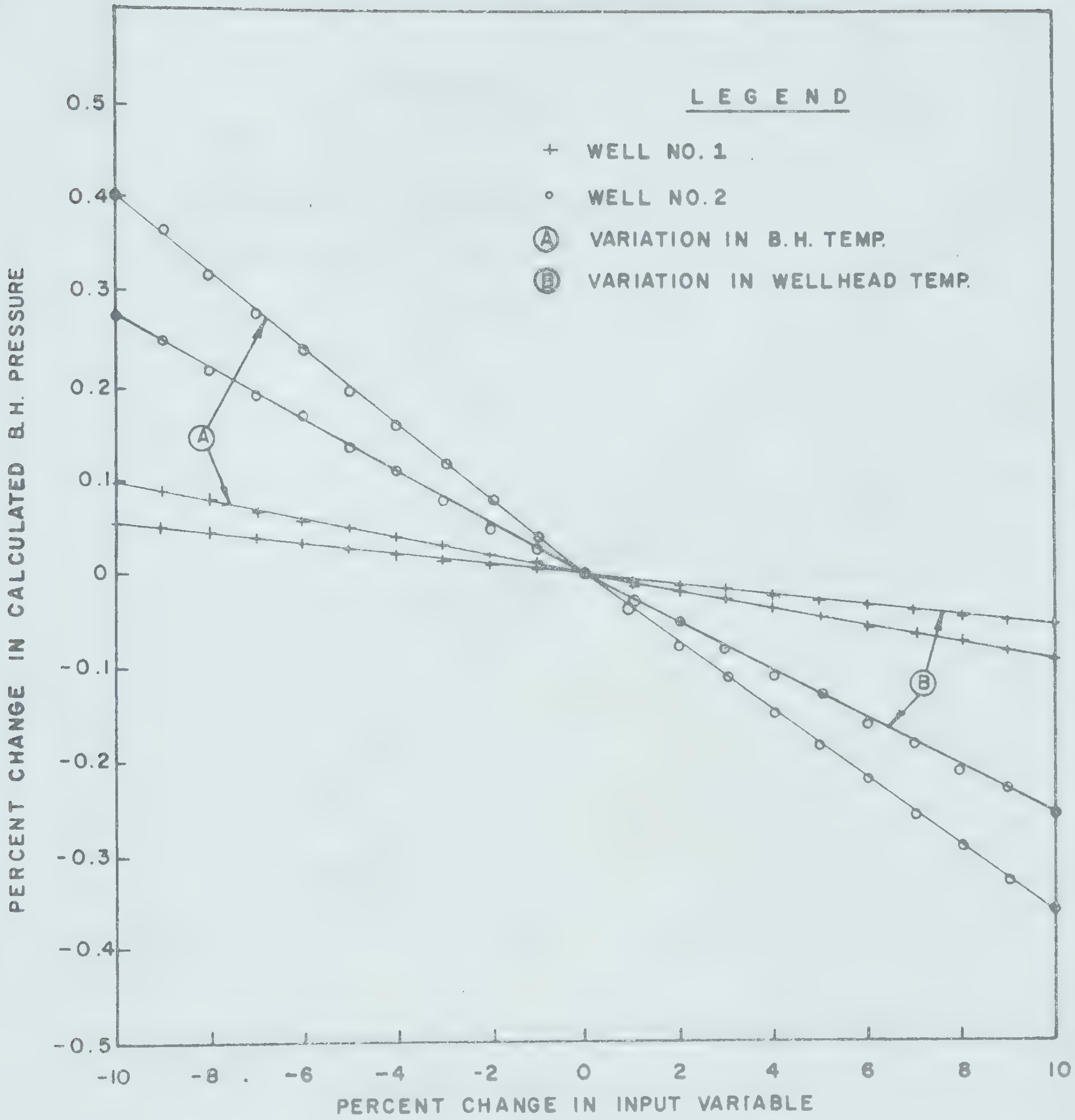

FIGURE 3 SENSITIVITY ANALYSIS OF AVERAGE TEMPERATURE AND AVERAGE COMPRESSIBILITY FACTOR METHOD - VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMPERATURE.

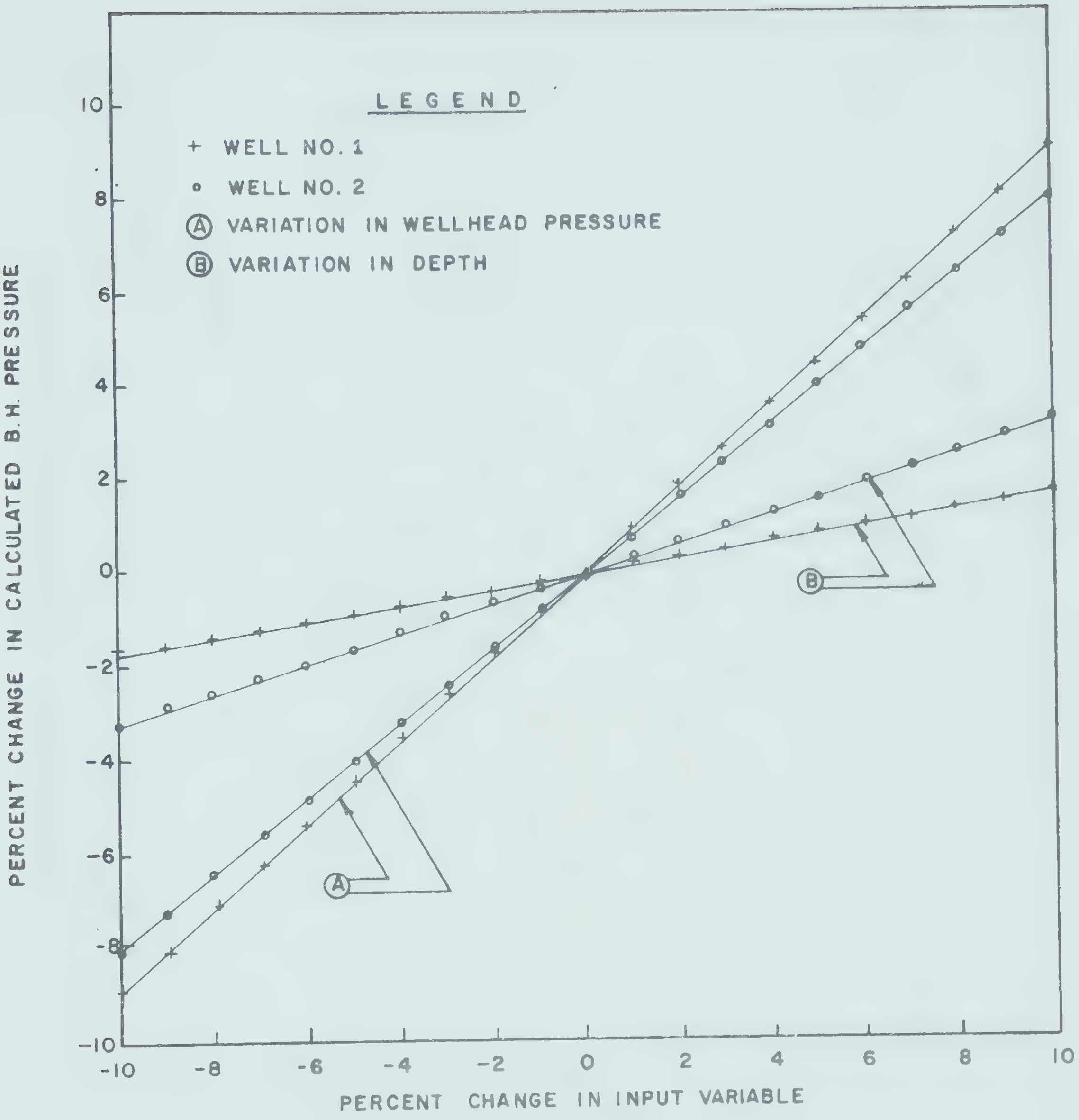

FIGURE 4 SENSITIVITY ANALYSIS OF AVERAGE DENSITY METHOD - VARIATION IN DEPTH AND WELLHEAD PRESSURE

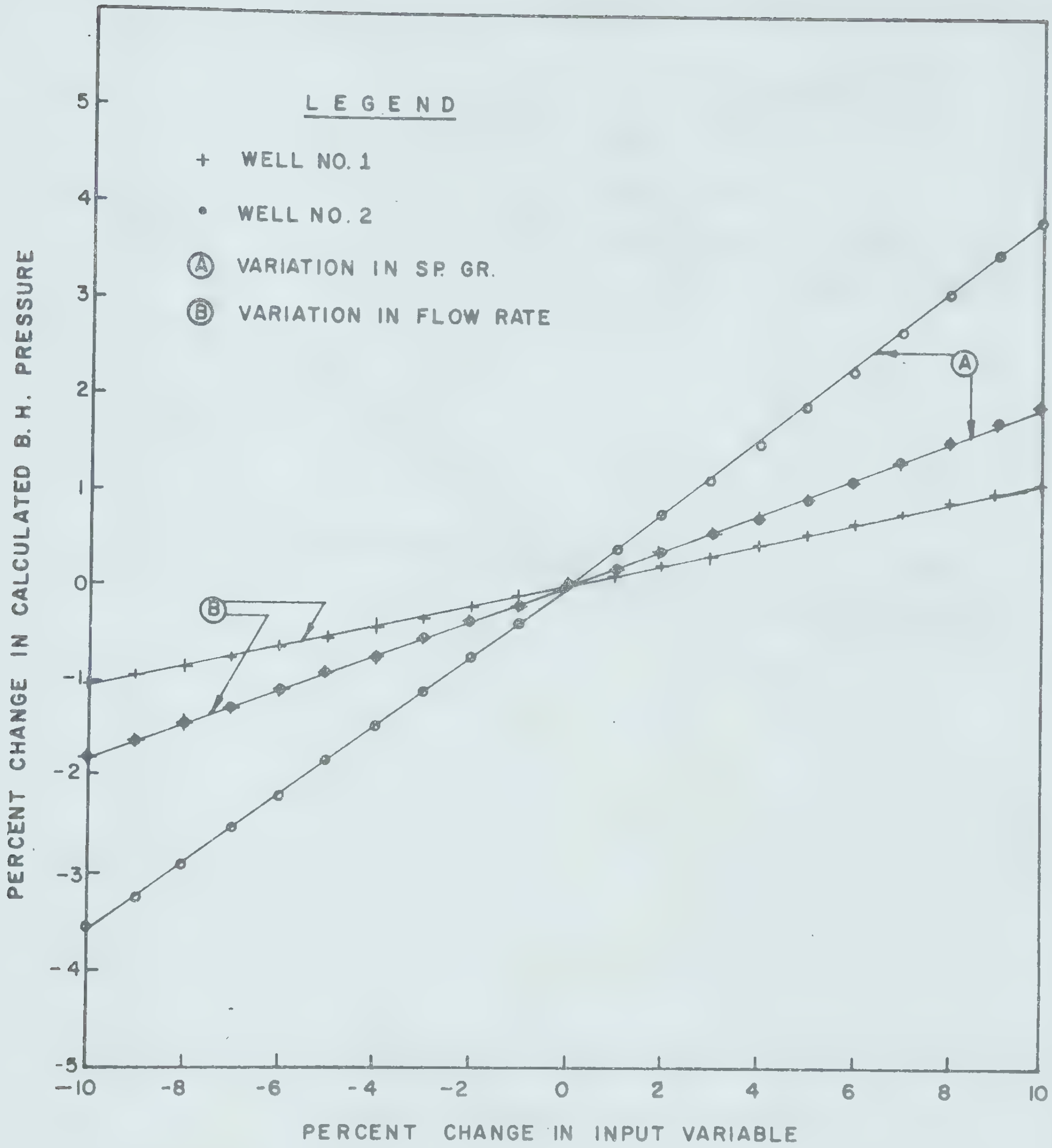

FIGURE 5 SENSITIVITY ANALYSIS OF AVERAGE DENSITY METHOD-VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

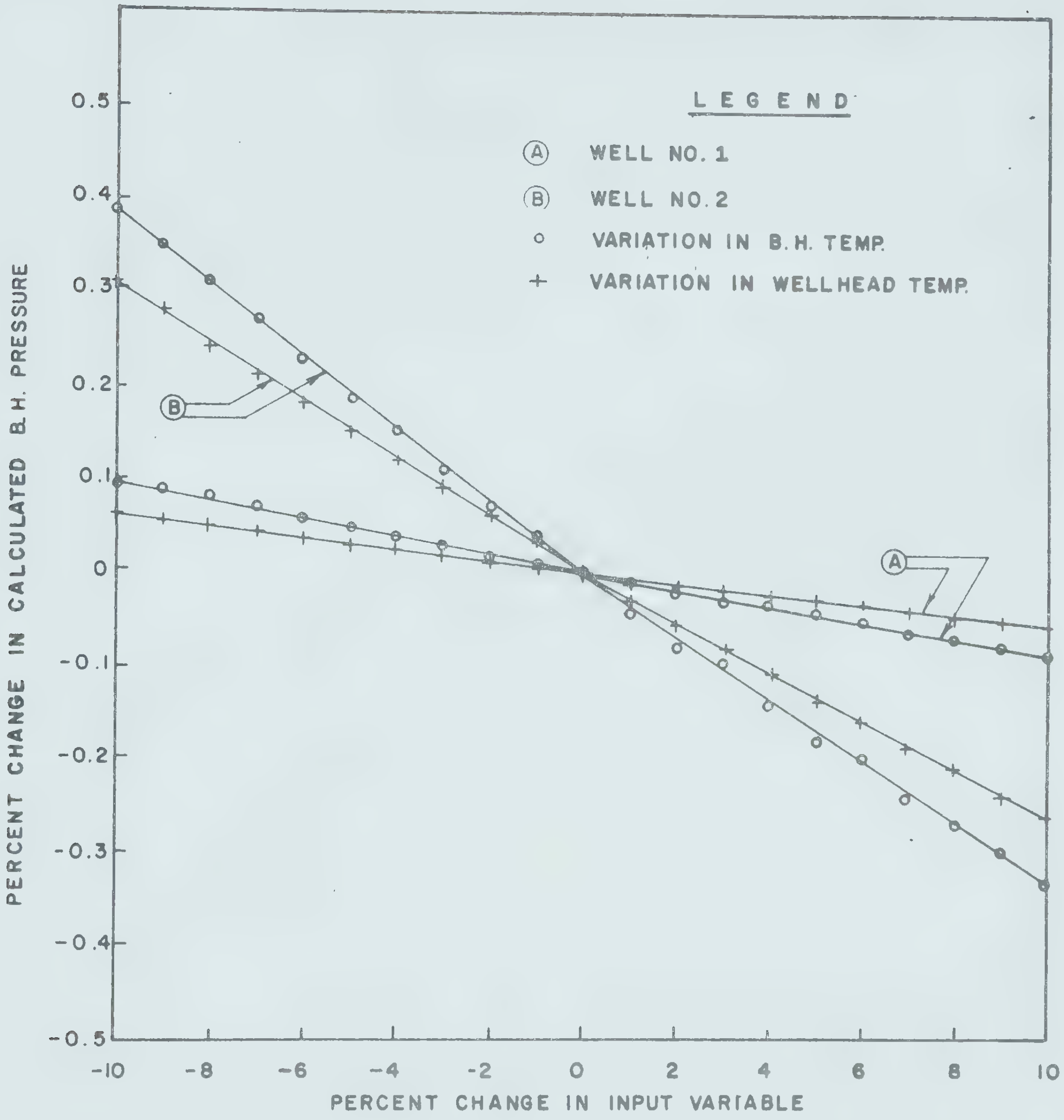

FIGURE 6 SENSITIVITY ANALYSIS OF AVERAGE DENSITY METHOD - VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMP.

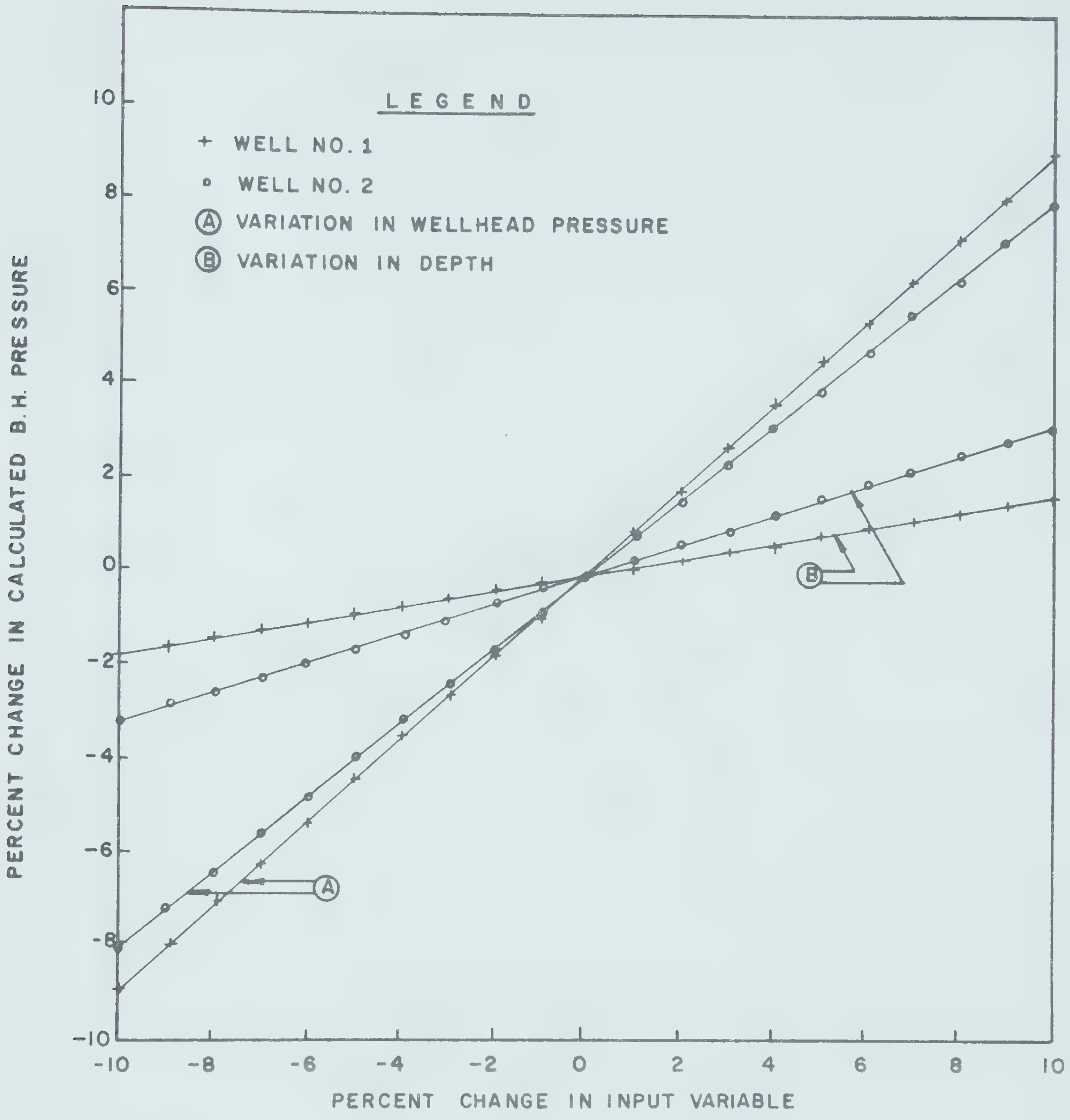

FIGURE 7 SENSITIVITY ANALYSIS OF CULLENDER AND SMITH METHOD-VARIATION IN DEPTH AND WELLHEAD PRESSURE

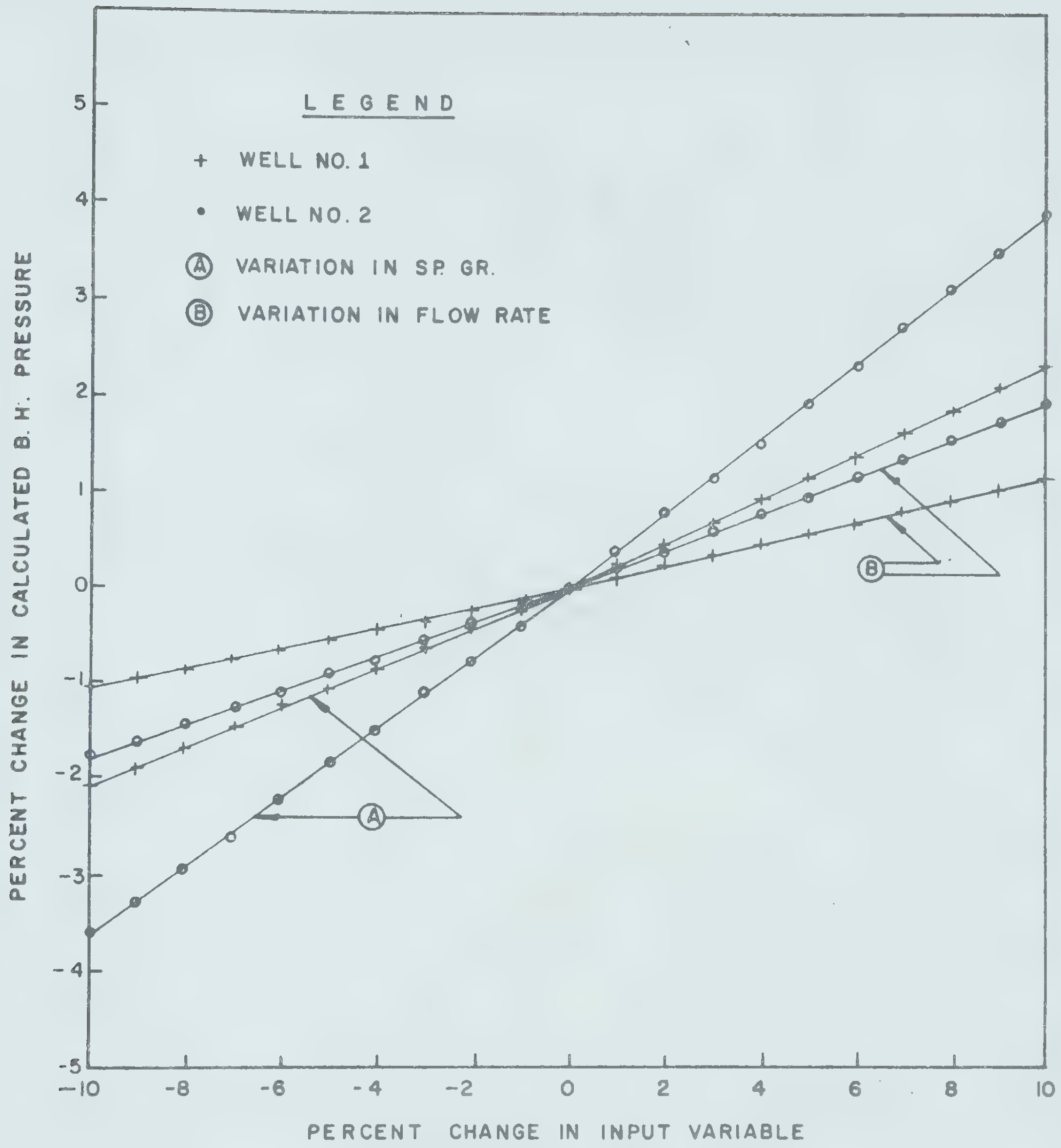

FIGURE 8 SENSITIVITY ANALYSIS OF CULLENDER AND SMITH METHOD-VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

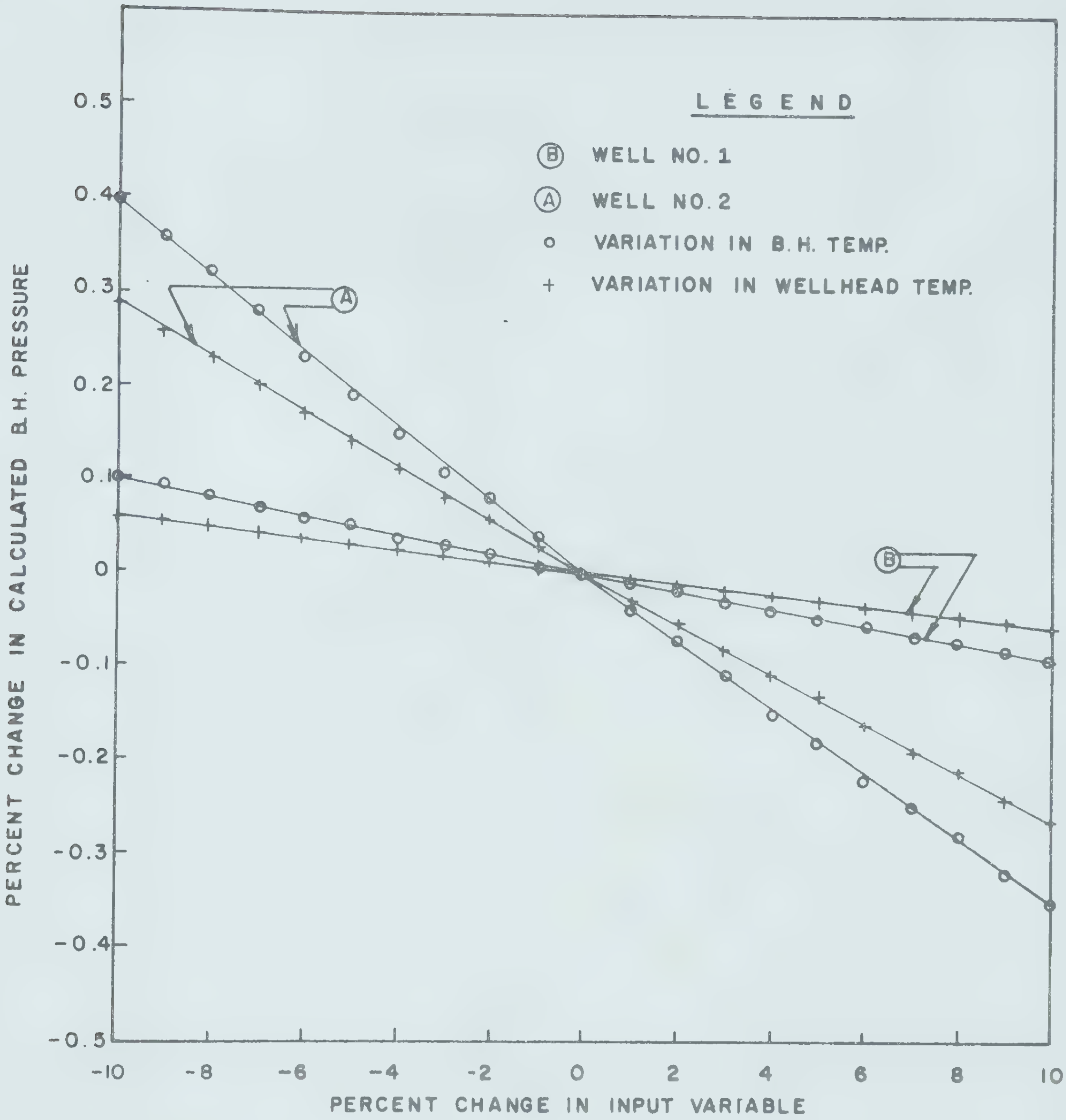

FIGURE 9 SENSITIVITY ANALYSIS OF CULLENDER AND SMITH METHOD-VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMP.

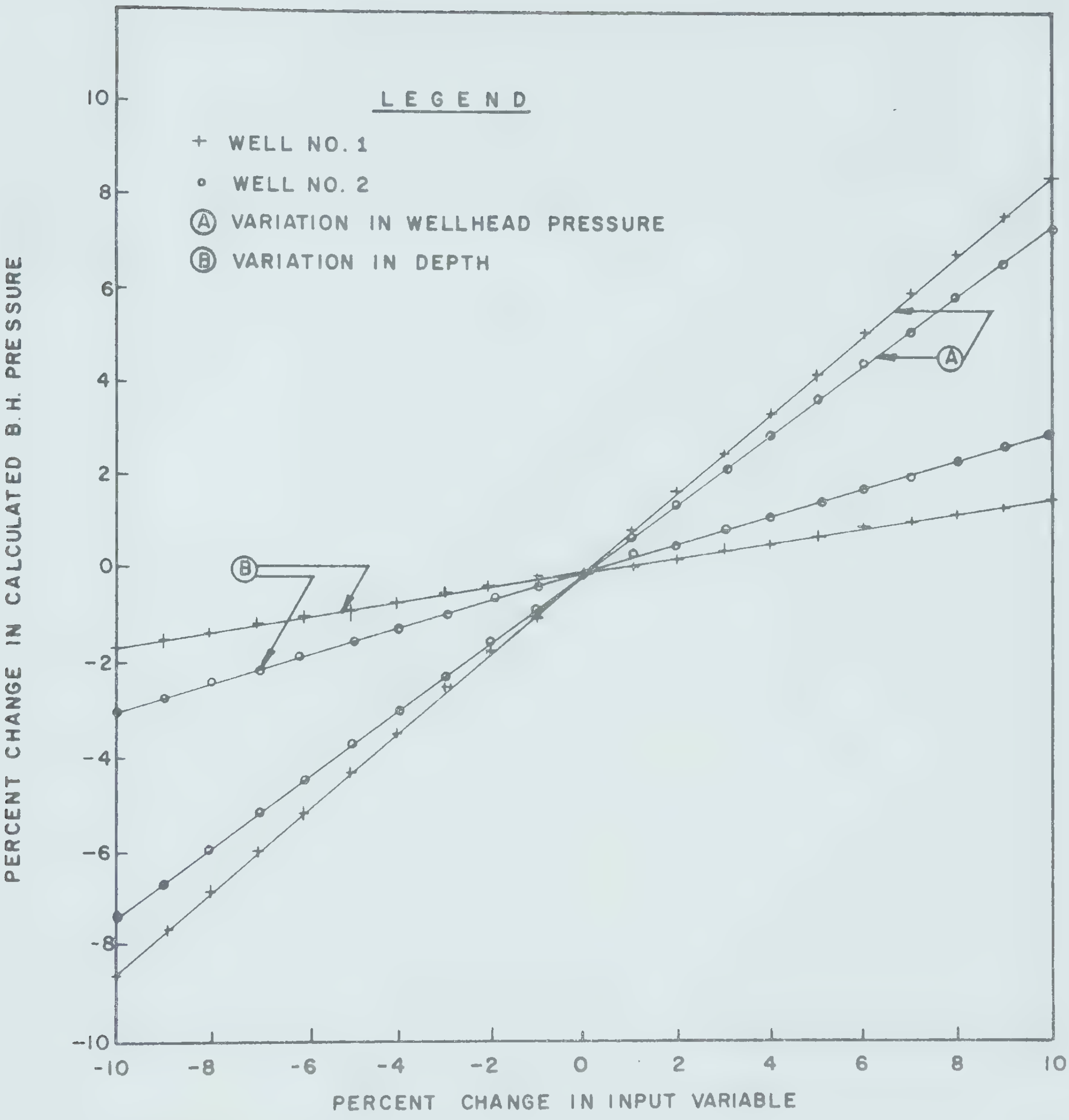

FIGURE 10 SENSITIVITY ANALYSIS OF POETTMANN METHOD-VARIATION IN DEPTH AND WELLHEAD PRESSURE

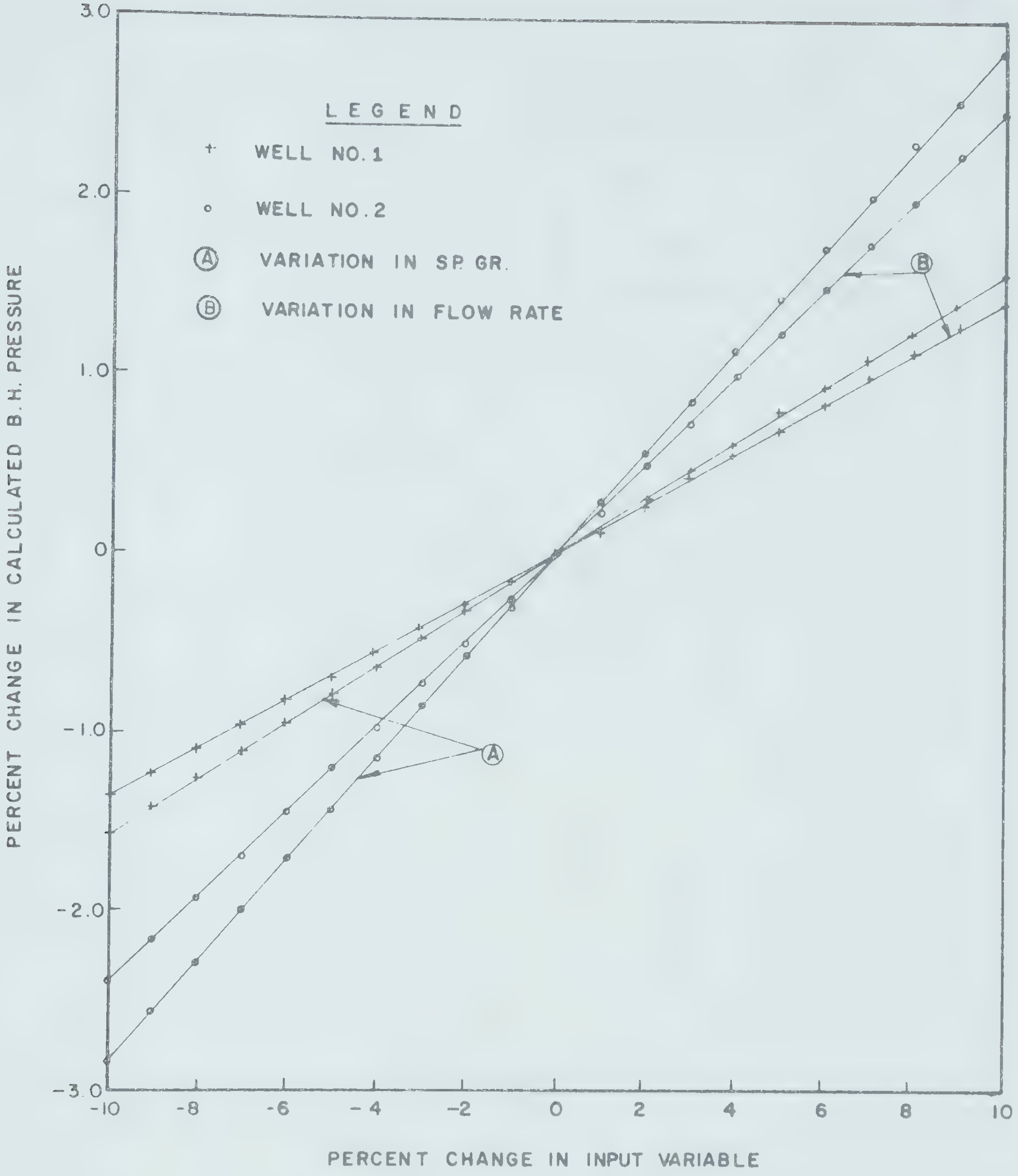

FIGURE 11 SENSITIVITY ANALYSIS OF POETTMAN METHOD — VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

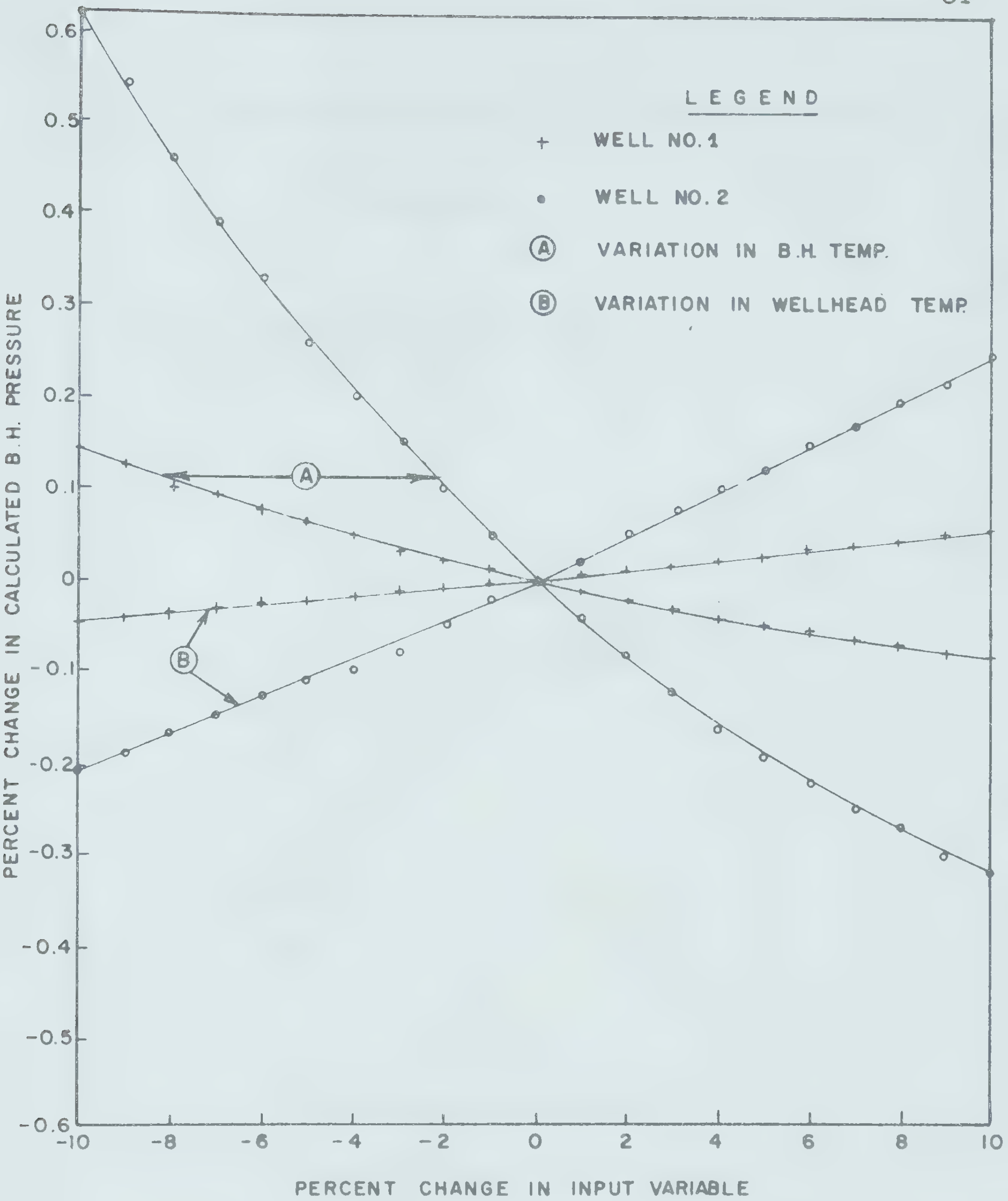

FIGURE 12 SENSITIVITY ANALYSIS OF POETTMANN METHOD—VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMPERATURE

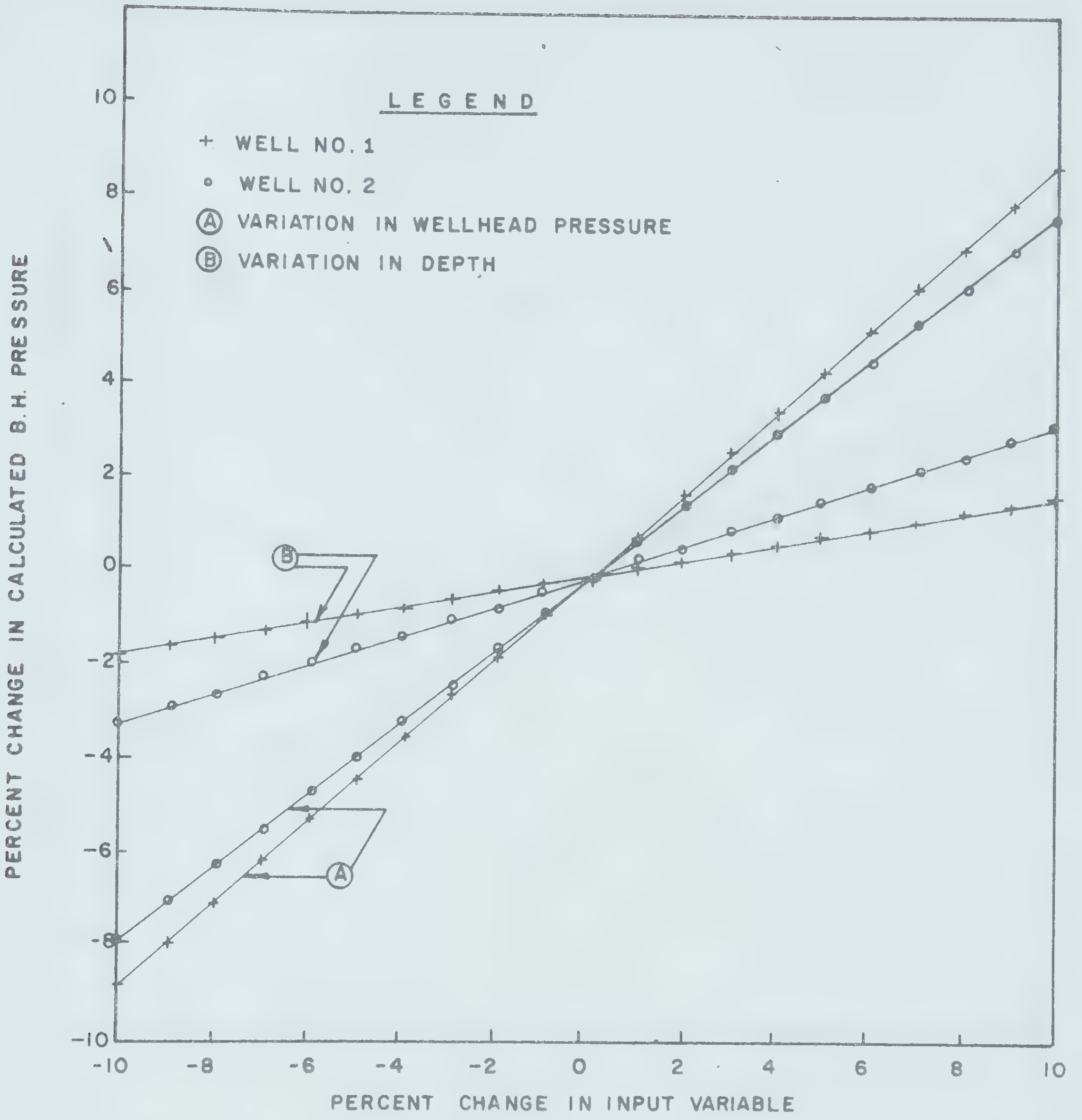

FIGURE 13 SENSITIVITY ANALYSIS OF SUKKAR AND CORNELL METHOD - VARIATION IN DEPTH AND WELLHEAD PRESSURE

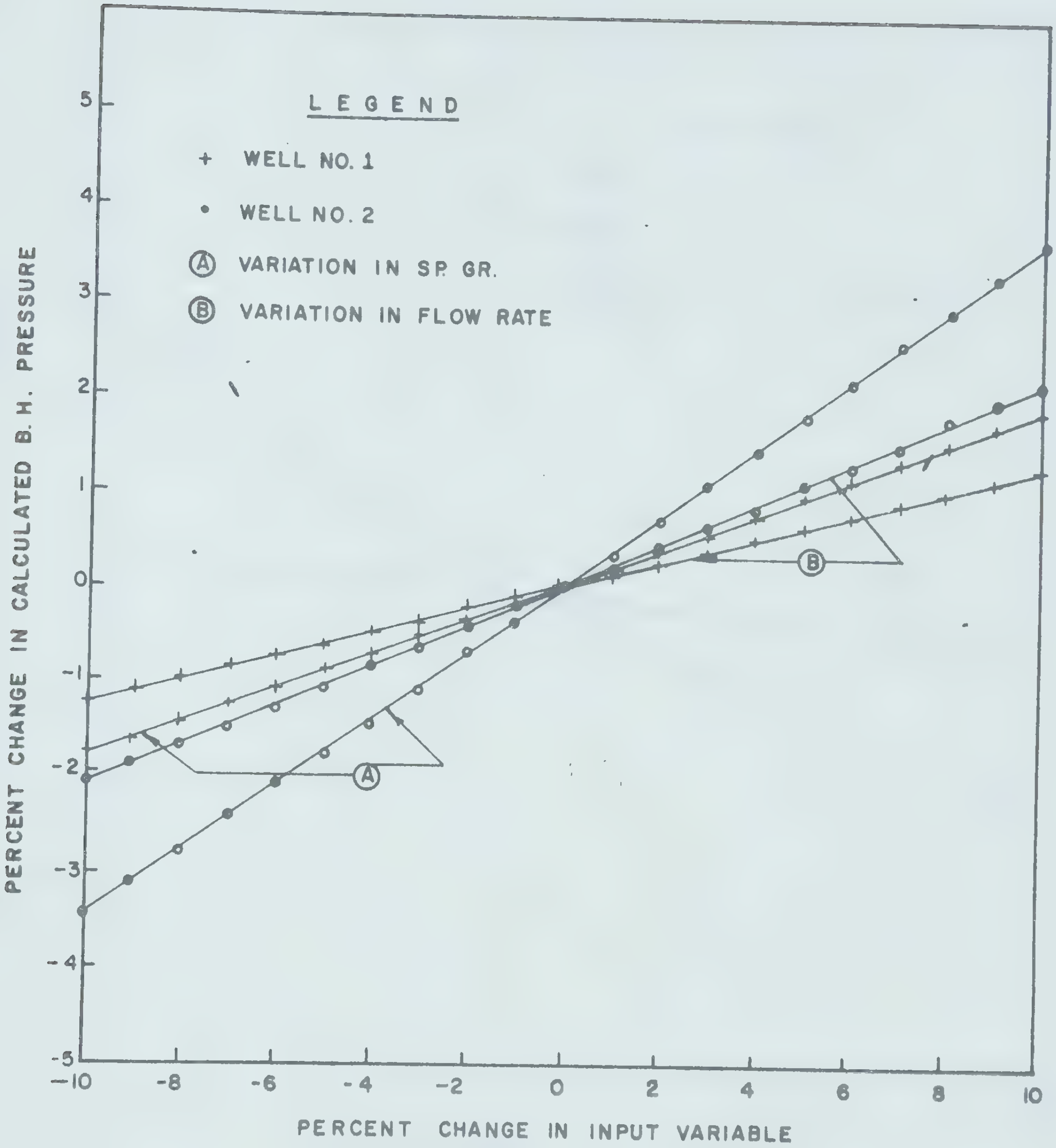

FIGURE 14 SENSITIVITY ANALYSIS OF SUKKAR AND CORNELL METHOD-VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

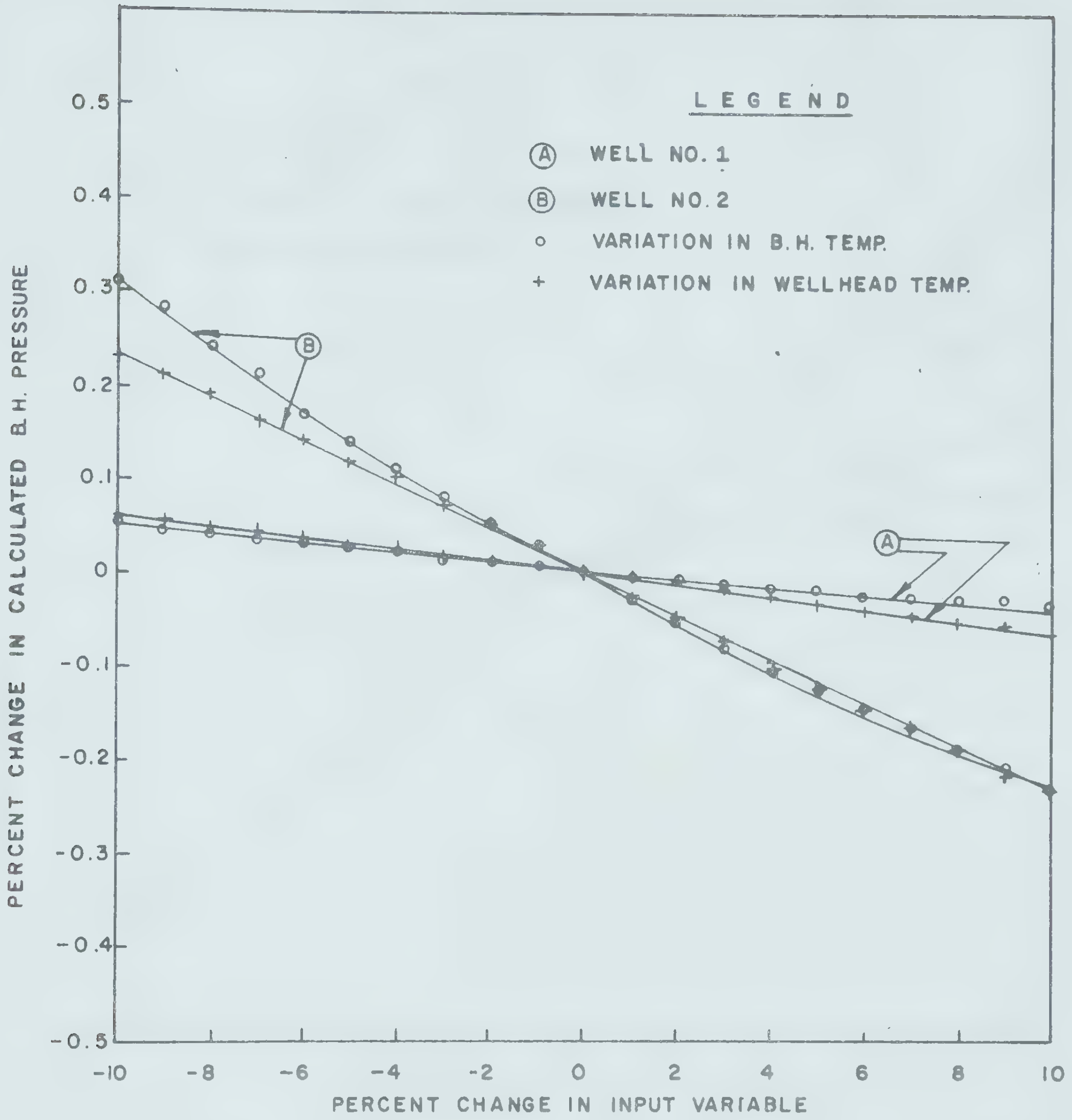

FIGURE 15 SENSITIVITY ANALYSIS OF SUKKAR AND CORNELL METHOD—VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMP.

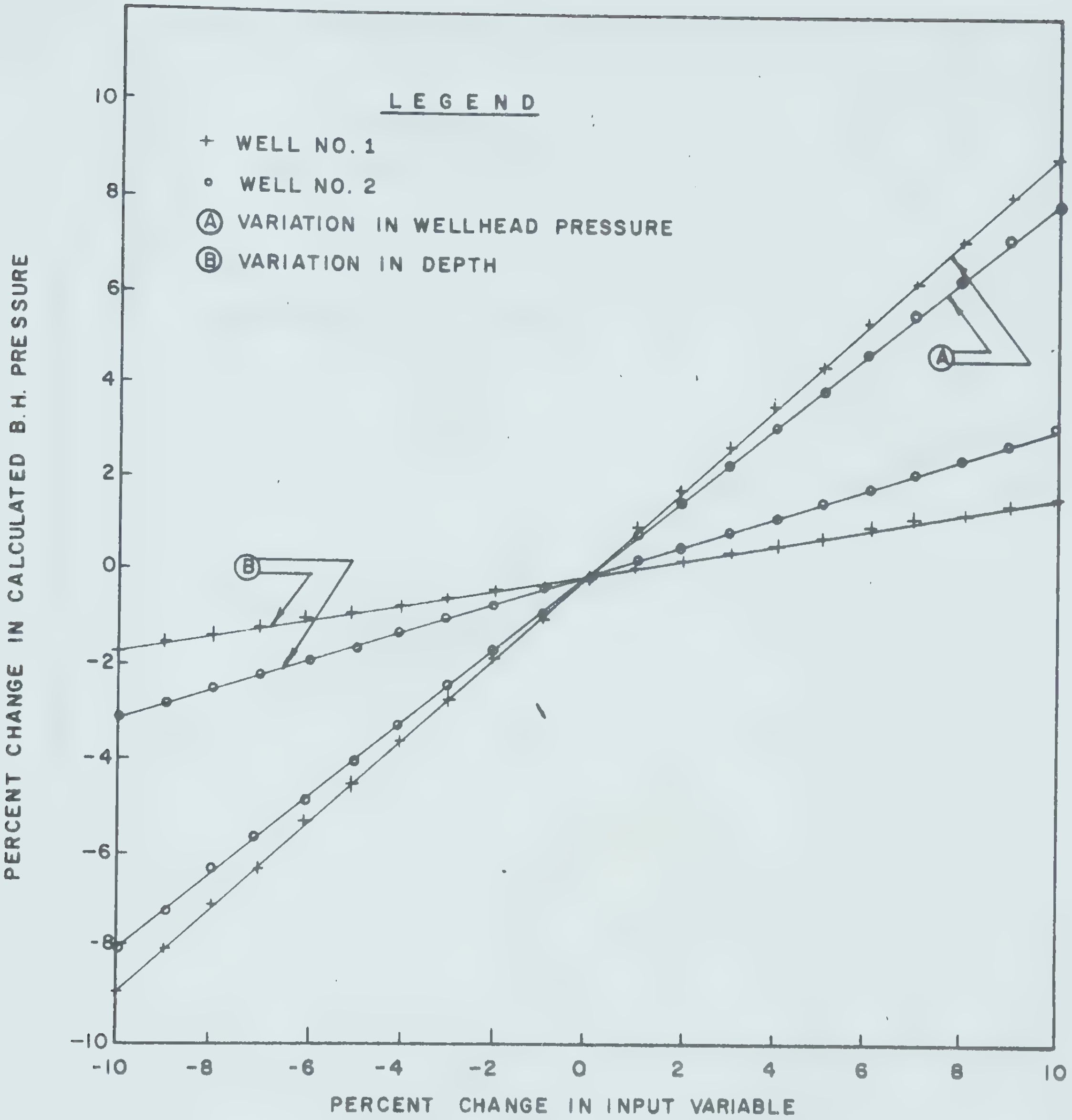

FIGURE 16 SENSITIVITY ANALYSIS OF DRANCHUK AND McFARLAND METHOD - VARIATION IN DEPTH AND WELLHEAD PRESSURE

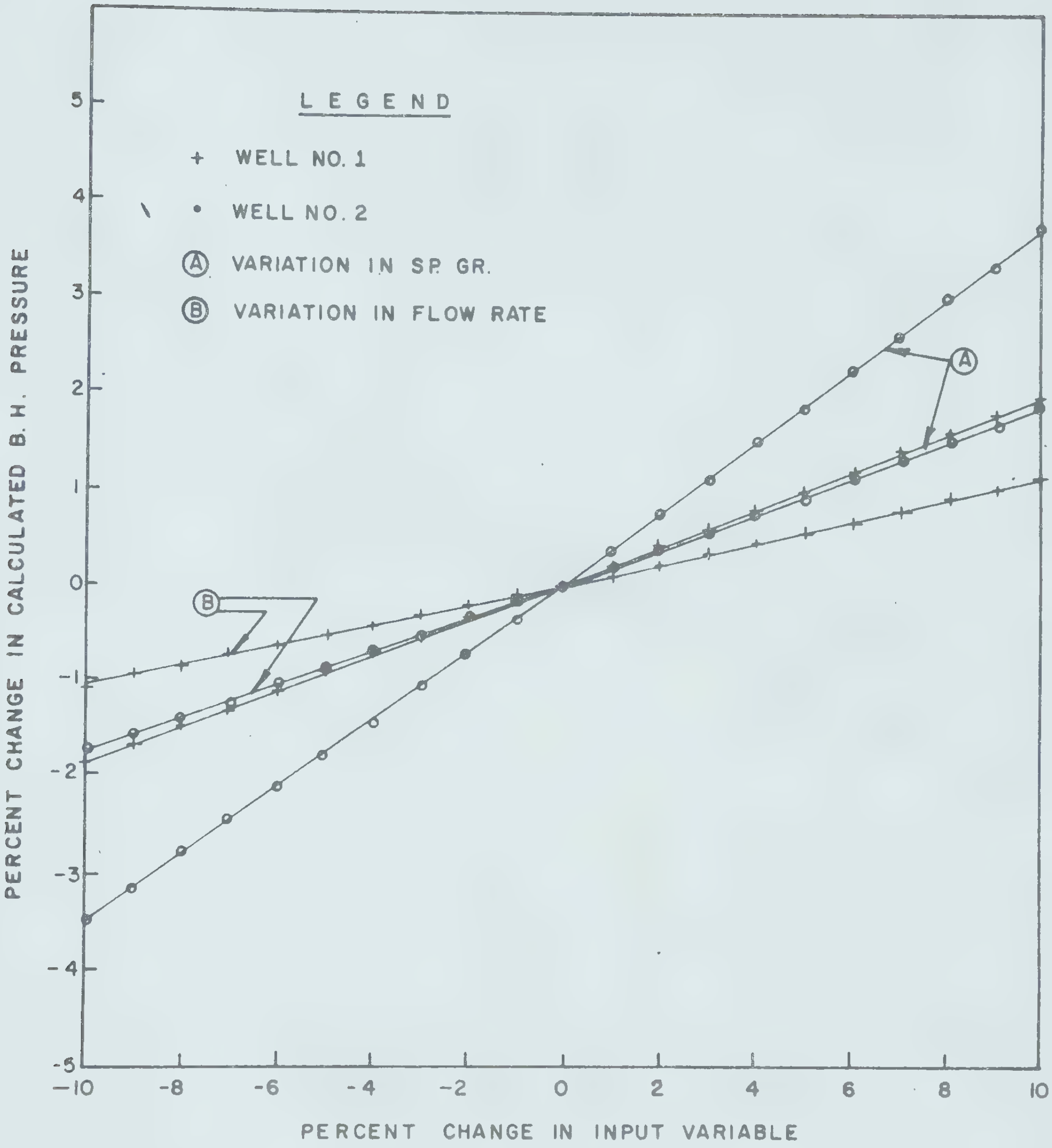

FIGURE 17 SENSITIVITY ANALYSIS OF DRANCHUK AND McFARLAND METHOD - VARIATION IN FLOW RATE AND SPECIFIC GRAVITY

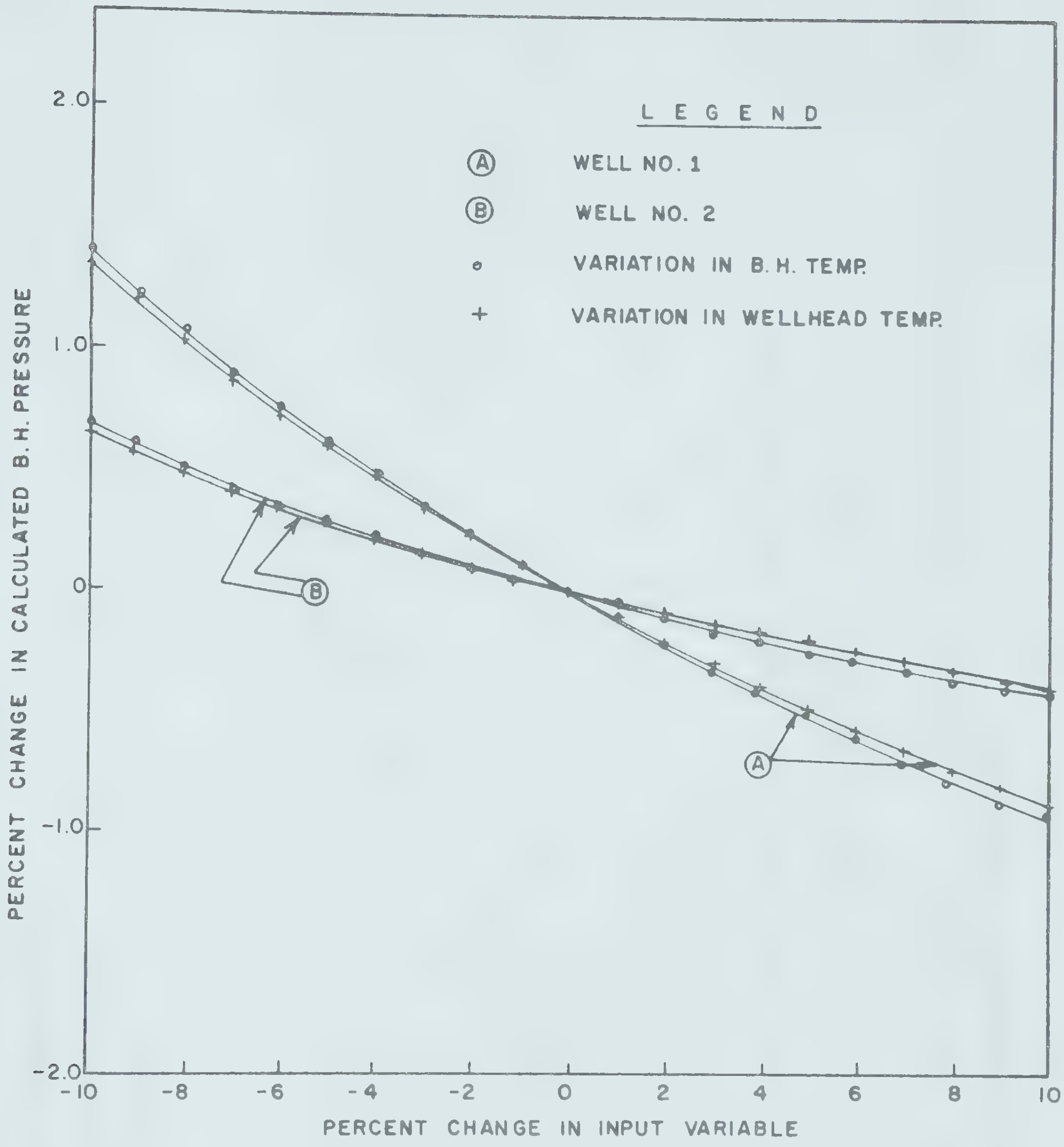

FIGURE 18 SENSITIVITY ANALYSIS OF DRANCHUK AND McFARLAND METHOD - VARIATION IN WELLHEAD AND BOTTOM-HOLE TEMP

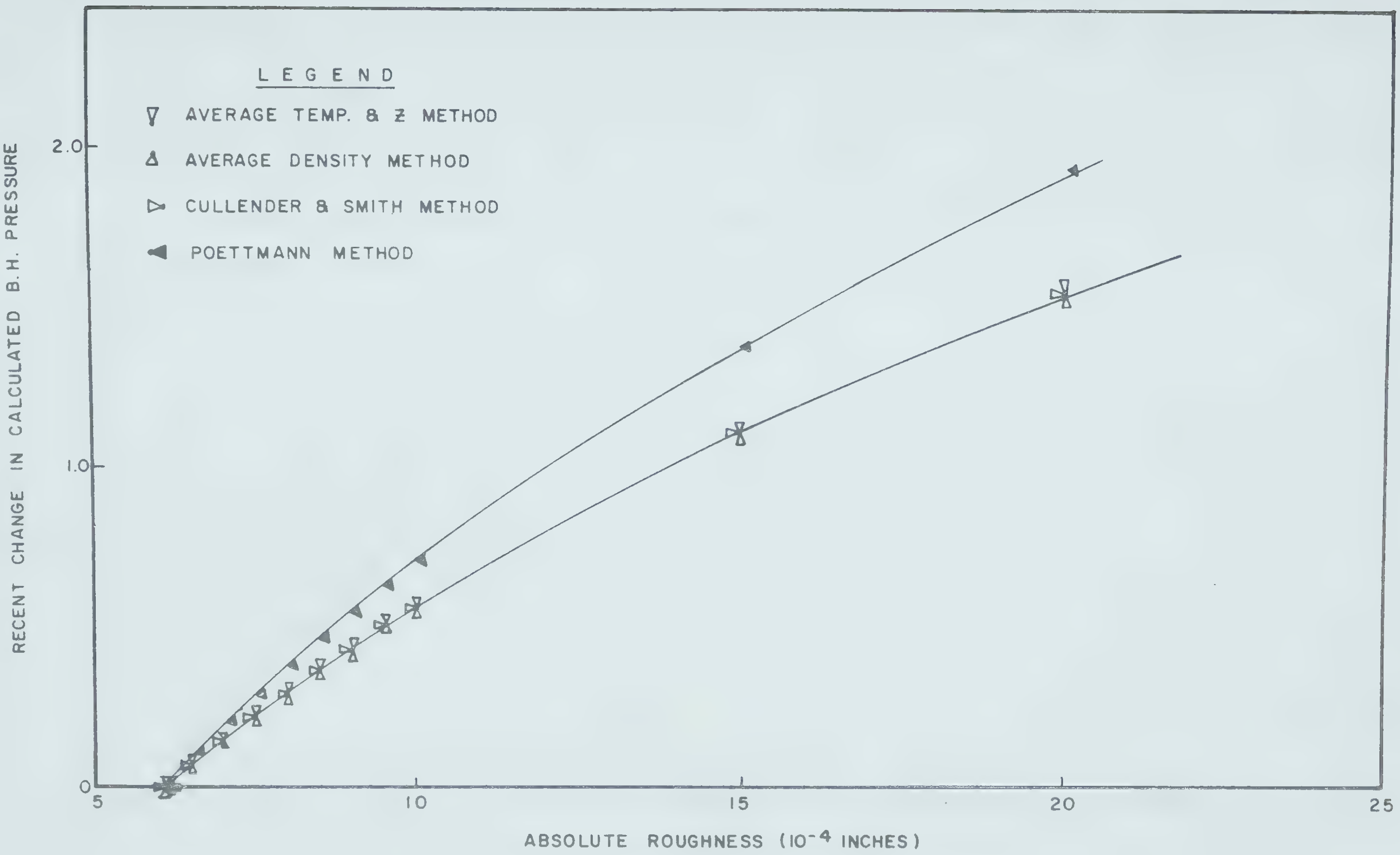

FIGURE 19 EFFECT OF VARIATION IN ABSOLUTE ROUGHNESS ON CALCULATED BOTTOM-HOLE PRESSURE — WELL NO. 1

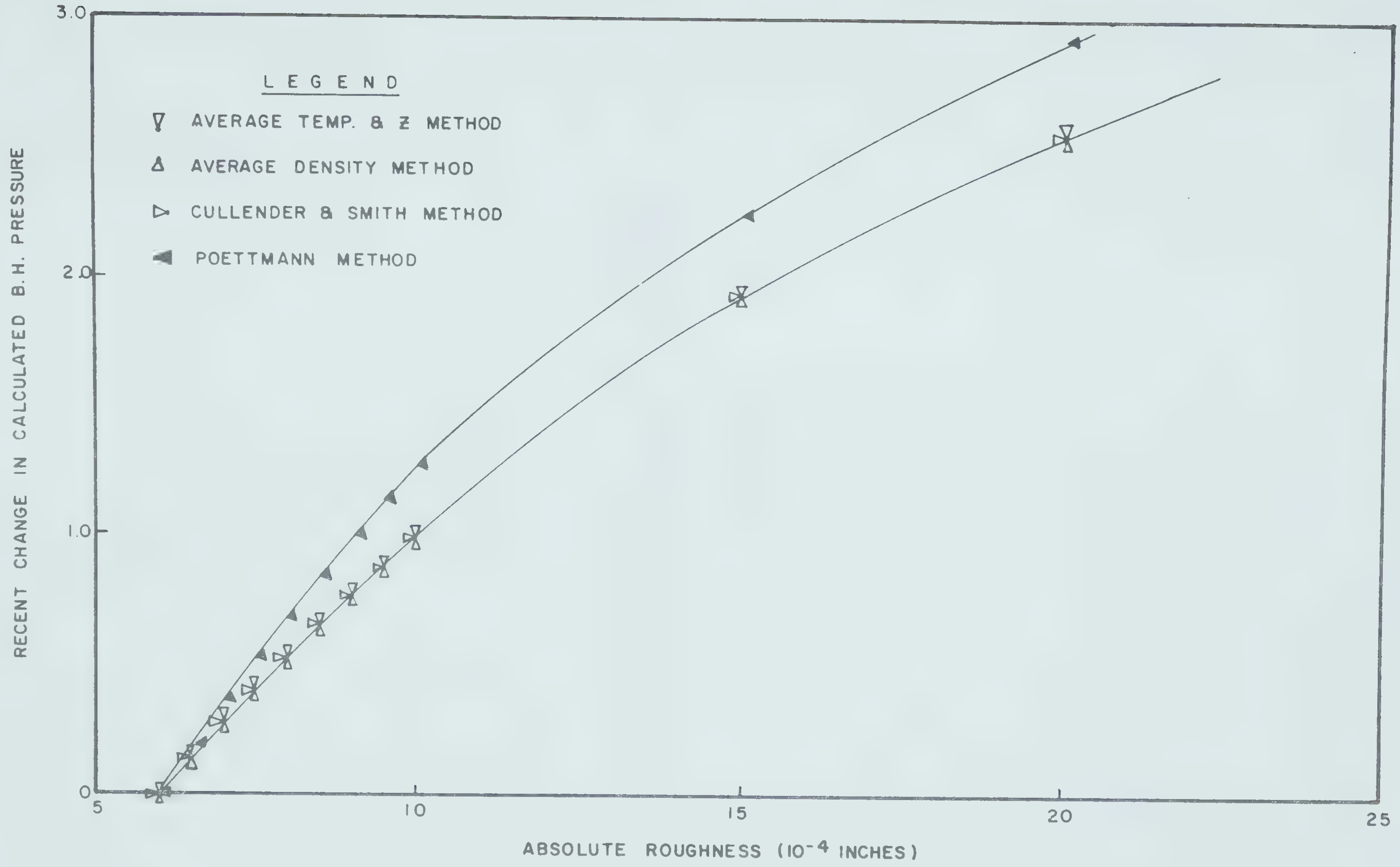

FIGURE 20 EFFECT OF VARIATION IN ABSOLUTE ROUGHNESS ON CALCULATED BOTTOM-HOLE PRESSURE — WELL NO. 2

APPENDIX B

EVALUATION OF POETTMANN AND SUKKAR-CORNELL INTEGRAL USING SIMPSON'S AND NEWTON'S 3/8 RULE

APPENDIX B

This integration scheme is listed in the computer program as subroutine QSF in Appendix D. This subroutine performs the integration of an equidistantly tabulated function by Simpson's rule. To compute the vector of integral values

$$\left.\begin{aligned} Z_i &= Z(x_i) = \int_a^{x_i} y(x)\,dx \\ \text{with} \qquad & \\ x_i &= a + (i-1)h \end{aligned}\right\} \quad i = 1, 2, \ldots, n$$

for a table of function values y_i $(i = 1, 2, \ldots, n)$ given at equidistant points $x_i = a + (i-1)h$, $(i = 1, 2, \ldots, n)$. Simpson's rule together with Newton's 3/8 rule or a combination of these rules is used. Local truncation error is of the order of h^5 in all cases with more than three points in the given table. Only Z_2 has a truncation error of the order h^4 if there are only three points in the given table. No action takes place if the table consists of less than three sample points.

The function is assumed continuous and differentiable (three or four times, depending on the rule used).

Formulas used in this subroutine (Z_j are integral values, y_i are function values) are:

$$Z_j = Z_{j-1} + \frac{h}{3}(1.25\, y_{j-1} + 2y_j - 0.25\, y_{j+1}) \;.\;.\;.\;.\;.\;.\; (1)$$

$$Z_j = Z_{j-2} + \frac{h}{3}(y_{j-2} + 4y_{j-1} + y_j) \quad \text{Simpson's Rule} \;.\;.\;.\; (2)$$

$$Z_j = Z_{j-3} + \frac{3}{8}h\,(y_{j-3} + 3y_{j-2} + 3y_{j-1} + y_j) \;.\;.\;.\;.\;.\;.\; (3)$$

Newton's 3/8 Rule

$$Z_j = Z_{j-5} + \frac{h}{3}(y_{j-5} + 3.875\, y_{j-4} + 2.625\, y_{j-3} + 2.625\, y_{j-2}$$

$$+ 3.875\, y_{j-1} + y_i) \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (4)$$

Combination of (2) and (3)

Sometimes formula (2) is used in the following form

$$Z_j = Z_{j+2} - \frac{h}{3}(y_j + 4y_{j+1} + y_{j+2})$$

Local truncation errors of formulas (1) - (4) are respectively:

$$R_1 = \frac{1}{24}h^4 y'''(\xi_1) \qquad [\xi_1 \text{ in}(x_{j-1}, x_{j+1})]$$

$$R_2 = -\frac{1}{90}h^5 y''''(\xi_2) \qquad [\xi_2 \text{ in}(x_{j-2}, x_j)]$$

$$R_3 = -\frac{3}{80}h^5 y''''(\xi_3) \qquad [\xi_3 \text{ in}(x_{j-3}, x_j)]$$

$$R_4 = -\frac{1}{144}h^5 y''''(\xi_4) \qquad [\xi_4 \text{ in}(x_{j-5}, x_j)]$$

APPENDIX C

DERIVATION OF ANALYTICAL SOLUTION

APPENDIX C

For the Average Temperature and Average Compressibility Factor Method and the Average Density Method, closed-form solution is available. Therefore analytical solutions can be obtained for these two cases. Generally, analytical solution is preferred if possible unless the final equation becomes too complicated and unmanageable.

This appendix gives the detailed derivations of the analytical solutions for the two cases above. The final equations can then be used to check the accuracy of the results obtained by numerical methods.

Average Temperature and Average Compressibility Factor Method

1. Effect of well depth, L.

The working equation is:

$$P_1 = e^c P_2^2 + \left[\frac{24.99\ Q_0^2\ G\langle T\rangle\langle Z\rangle fL(e^c - 1)}{d^5 c}\right]^{\frac{1}{2}} \qquad (1)$$

where

$$c = \frac{2GL}{53.34\langle T\rangle\langle Z\rangle}$$

Let $k_1 = P_2^2$

$$k_2 = \frac{2G}{53.34\langle T\rangle\langle Z\rangle}$$

and $k_3 = \dfrac{25\ Q_0^2\langle T\rangle\langle Z\rangle Gf}{d^5}$

Equation (1) can be written as follows,

$$P_1 = \left(k_1\ e^{k_2 L} + \frac{k_3}{k_2}\ e^{k_2 L} - \frac{k_3}{k_2}\right)^{\frac{1}{2}} \qquad (2)$$

By differentiating Equation (2) with respect to L, one obtains,

$$\frac{dP_1}{dL} = \frac{k_1\ k_2\ e^{k_2 L} + k_3\ e^{k_2 L}}{2P_1} \tag{3}$$

By substituting the values of k_1, k_2, and k_3 into Equation (3) one obtains

$$\frac{dP_1}{dL} = \frac{e^{\left(\frac{2GL}{53.34<T><Z>}\right)}\left[\frac{2GP_2^2}{53.34<T><Z>} + \frac{24.99\ Q_0^2\ Gf<T><Z>}{d^5}\right]}{2P_1} \tag{4}$$

2. Effect of flow rate, Q_0.

Let $k_1 = e^c\ P_2^2$

and $k_2 = \frac{25\ GLf<T><Z>(e^c - 1)}{d^5 c}$

where

$$c = \frac{2GL}{53.34<T><Z>}$$

Substituting k_1 and k_2 into Equation (1) obtains

$$P_1 = (k_1 + k_2 Q_0^2)^{\frac{1}{2}} \tag{5}$$

By differentiating Equation (5) with respect to Q_0, one obtains,

$$\frac{dP_1}{dQ_0} = \frac{k_2 Q_0}{(k_1 + k_2 Q_0^2)^{\frac{1}{2}}} \tag{6}$$

By substituting Equation (5) into Equation (6), one obtains,

$$\frac{dP_1}{dQ_0} = \frac{k_2 Q_0}{P_1} = \frac{25\ GLfQ_0<T><Z>\ (e^{\frac{2GL}{53.34<T><Z>}} - 1)}{P_1} \tag{7}$$

3. Effect of wellhead pressure, P_2.

Let $k_1 = \dfrac{25\ Q_0^2\ G\langle T\rangle\langle Z\rangle fL(e^C - 1)}{d^5 c}$

where

$$c = \frac{2GL}{53.34\langle T\rangle\langle Z\rangle}$$

Substitute k into Equation (1) to obtain

$$P_1 = (e^C\ P_2^2 + k_2)^{\frac{1}{2}} \tag{8}$$

By differentiating Equation (8) with respect to P_2, one obtains,

$$\frac{dP_1}{dP_2} = \frac{e^C\ \ P_2}{(e^C\ P_2^2 + k_2)^{\frac{1}{2}}} \tag{9}$$

By substituting Equation (8) and value of c into Equation (9), one obtains

$$\frac{dP_1}{dP_2} = \frac{P_2}{P_1}\ e^{\left(\frac{2GL}{53.34\langle T\rangle\langle Z\rangle}\right)} \tag{10}$$

4. Effect of bottom-hole temperature, T_1 and wellhead temperature, T_2.

Let $k_1 = P_2^2$

$k_2 = \dfrac{2GL}{53.34}$

and $k_3 = \dfrac{25\ Q_0^2\ GfL}{d^5}$

Substituting k_1, k_2, and k_3 into Equation (1), one obtains,

$$P_1 = \left[k_1\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} + \frac{\left(k_3\, \langle T\rangle\langle Z\rangle \quad e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} - 1\right)}{k_2 \left(\frac{1}{\langle T\rangle\langle Z\rangle}\right)} \right]^{\frac{1}{2}} \qquad (11)$$

Equation (11) can be rearranged to yield

$$P_1 = k_1\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} + \frac{k_3}{k_2} \langle T\rangle^2\langle Z\rangle^2\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} - \frac{k_3}{k_2} \langle T\rangle^2\langle Z\rangle^2 \qquad (12)$$

Since

$$\langle T\rangle = \frac{T_1 + T_2}{2}$$

and

$$\frac{d\langle T\rangle}{dT_1} = \frac{1}{2} = \frac{d\langle T\rangle}{dT_2}$$

therefore

$$\frac{\partial\langle Z\rangle}{\partial T_1} = \frac{\partial\langle Z\rangle}{\partial\langle T\rangle} \cdot \frac{d\langle T\rangle}{dT_1} = \frac{1}{2}\,\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}$$

By differentiating Equation (12) with respect to T_1, one obtains

$$\begin{aligned}
\frac{dP_1}{dT_1} = \frac{dP_1}{dT_2} = \Bigg[& \frac{-k_1\ k_2}{\partial\langle Z\rangle\langle T\rangle^2}\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} - \frac{-k_1\ k_2}{\partial\langle Z\rangle^2\langle T\rangle}\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} \\
& + \frac{k_3}{k_2} \langle Z\rangle\langle T\rangle^2\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} + \frac{k_3}{k_2} \langle Z\rangle^2\langle T\rangle\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} \\
& - \frac{k_2}{2} \langle Z\rangle\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} - \frac{k_3}{2} \langle T\rangle\, e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} \\
& - \frac{k_3}{2} \langle Z\rangle\langle T\rangle^2 \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} - \frac{k_3}{k_2} \langle Z\rangle^2\langle T\rangle \Bigg] \Big/ (2P_1)
\end{aligned}$$

$$\ldots\ldots(13)$$

Equation (13) can be rearranged to yield

$$\begin{aligned}\frac{dP_1}{dT_1} = \frac{dP_1}{dT_2} = \Bigg[e^{\frac{k_2}{\langle T\rangle\langle Z\rangle}} \Bigg(& \frac{-k_1\ k_2}{\partial\langle Z\rangle\langle T\rangle^2} - \frac{-k_1\ k_2}{\partial\langle Z\rangle\langle T\rangle^2} \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} \\ & + \frac{k_3}{k_2} \langle Z\rangle\langle T\rangle^2 \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} + \frac{k_3}{k_2} \langle Z\rangle^2\langle T\rangle \\ & - \frac{k_3}{2} \langle Z\rangle^2\langle T\rangle - \frac{k_3}{2} \langle Z\rangle - \frac{k_3}{2} \langle T\rangle \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} \Bigg) \\ & - \frac{k_3}{2} \langle Z\rangle\langle T\rangle^2 \left(\frac{\partial\langle Z\rangle}{\partial\langle T\rangle}\right)_{\bar{P}_r} - \frac{k_3}{k_2} \langle Z\rangle^2\langle T\rangle \Bigg] / (2P_1) \\ & \qquad \ldots\ldots(14)\end{aligned}$$

5. Effect of gas gravity, G.

Let $k_1 = P_2^2$

$k_2 = \frac{2L}{53.34\langle T\rangle\langle Z\rangle}$

and $k_3 = \frac{25\ Q_0^2\langle T\rangle\langle Z\rangle fL}{d^5}$

Substituting k_1, k_2, and k_3 into Equation (1), one obtains,

$$P_1 = \left(k_1\ e^{k_2 G} + \frac{k_3}{k_2}\ e^{k_2 G} - \frac{k_3}{k_2} \right)^{\frac{1}{2}} \tag{15}$$

By differentiating Equation (15) with respect to G, one obtains,

$$\frac{dP_1}{dG} = \frac{e^{k_2 G}(k_1\ k_2 + k_3)}{2P_1} \tag{16}$$

Substitute value of k_1, k_2, and k_3 to obtain

$$\frac{dP_1}{dG} = \frac{e^{\frac{2GL}{53.34<T><Z>}}\left(\frac{2LP_2^2}{53.34<T><Z>} + \frac{25\ Q_0^2<T><Z>fL}{d^5}\right)}{2P_1} \qquad (17)$$

Average Density Method

1. Effect of well depth, L.

The working equation is:

$$P_1 = P_2 + 9.374 \times 10^{-3}\ GL\left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right) + \frac{24.99\ Q_0^2\ GLf}{d^5}\left(\frac{1}{\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}}\right) \qquad (18)$$

Let $k_1 = 9.374 \times 10^{-3}\ G$

$$k_2 = \frac{24.99\ Q_0^2\ Gf}{d^5}$$

and $k_3 = \frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}$

Substituting k_1, k_2, and k_3 into Equation (18) yields

$$P_1 = P_2 + k_1\ k_3\ L + \frac{k_2}{k_3}L \qquad (19)$$

Equation (19) can be differentiated with respect to L to give

$$\frac{dP_1}{dL} = k_1\ k_3 + \frac{k_1\ L\ \frac{dP_1}{dL}}{T_1Z_1} + \frac{k_2}{k_3} - \frac{k_2\ L}{k_3^2\ T_1Z_1}\frac{dP_1}{dL} \qquad (20)$$

By grouping terms and substituting the value of k_1, k_2, and k_3, Equation (20) can be rewritten as

$$\frac{dP_1}{dL} = \frac{9.374 \times 10^{-3}\ G \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right) + \frac{24.99\ Q_o^2\ Gf}{d^5 \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)}}{1 + \frac{1}{T_1Z_1}\left(\frac{24.99\ Q_o^2\ GfL}{d^5 \left(\frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}\right)^2}\right) - 9.374 \times 10^{-3}\ GL} \qquad (21)$$

2. Effect of flow rate, Q_o.

Let $k_1 = 9.374 \times 10^{-3}\ GL$

$k_2 = \frac{24.99\ GLf}{d^5}$

and $k_3 = \frac{P_1}{T_1Z_1} + \frac{P_2}{T_2Z_2}$

Substituting k_1, k_2, and k_3 into Equation (18) yields

$$P_1 = P_2 + k_1\ k_3 + \frac{k_2\ Q_o^2}{k_3} \qquad (22)$$

Equation (22) can be differentiated with respect to Q_o to give

$$\frac{dP_1}{dQ_o} = \frac{k_1 \frac{dP}{dQ_o}}{T_1Z_1} + 2\ \frac{k_2}{k_3}\ Q_o - \frac{k_2\ Q_o^2}{k_3^2}\ \frac{\frac{dP_1}{dQ_o}}{Z_1T_1} \qquad (23)$$

By grouping terms and substituting the value of k_1, k_2 and k_3 into Equation (23), one obtains

$$\frac{dP_1}{dQ_o} = \frac{49.98\left(\dfrac{9.374 \times 10^{-3}\ GL}{\dfrac{P_1}{T_1Z_1} + \dfrac{P_2}{T_2Z_2}}\right)}{1 - \dfrac{9.374 \times 10^{-3}\ GL}{T_1Z_1} + \dfrac{24.99\ GLfQ_o^2}{T_1Z_1\left(\dfrac{P_1}{T_1Z_1} + \dfrac{P_2}{T_2Z_2}\right)^2}} \tag{24}$$

3. Effect of wellhead pressure, P_2.

Let $k_1 = 9.374 \times 10^{-3}\ GL$

$$k_2 = \frac{24.99\ Q_o^2\ GLf}{d^5}$$

and $k_3 = \dfrac{P_1}{T_1Z_1} + \dfrac{P_2}{T_2Z_2}$

Substituting k_1, k_2, and k_3 into Equation (18) yields

$$P_1 = P_2 + k_1\ k_3 + \frac{k_2}{k_3} \tag{25}$$

Equation (25) can be differentiated with respect to P_2 to give

$$\frac{dP_1}{dP} = 1 + k_1\frac{dk_3}{dP_2} - \frac{k_2}{k_3}\frac{dk_3}{dP_2} \tag{26}$$

Since

$$\frac{dk_3}{dP_2} = \frac{\dfrac{dP_1}{dP_2}}{T_1Z_1} + \frac{1}{T_1Z_1} - \frac{P_2}{T_2Z_2^2}\left(\frac{\partial Z_2}{\partial P_2}\right)_{T_2} \tag{27}$$

Equation (26) can be rewritten as

$$\frac{dP_1}{dP_2} = 1 + \left(k_1 - \frac{k_2}{k_3}\right)\left[\frac{\dfrac{dP_1}{dP_2}}{T_1Z_1} + \frac{1}{T_2Z_2} - \frac{P_2}{T_2Z_2^2}\left(\frac{\partial Z_2}{\partial P_2}\right)_{T_2}\right] \tag{28}$$

By grouping terms and substituting the value of k_1, k_2, and k_3 into Equation (28), one obtains

$$\frac{dP_1}{dP_2} = \frac{1 - \frac{1}{T_2 Z_2}\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_o^2\ GLf}{d^5\left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}\right)}\right)\left[1 - \frac{P_2}{Z_2}\left(\frac{\partial Z_2}{P_2}\right)_{T_2}\right]}{1 - \frac{1}{T_1 Z_1}\left(9.374 \times 10^{-3}\ GL - \frac{24.99\ Q_o^2\ GLf}{d^5\left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}\right)}\right)} \quad . . . . . (29)$$

4. Effect of bottom-hole temperature, T_1 and wellhead temperature, T_2.

Let $k_1 = 9.374 \times 10^{-3}\ GL$

$$k_2 = \frac{24.99\ Q_o^2\ GLf}{d^5}$$

and $$k_3 = \frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}$$

By differentiating k_3 with respect to T_1, one obtains

$$\frac{dk_3}{dT_1} = \frac{\frac{dP_1}{dT_1}}{T_1 Z_1} - \frac{P_1}{T_1^2 Z_1} - \frac{P_1}{T_1 Z_1^2}\left(\frac{\partial Z}{\partial T_1}\right)_{P_1} \qquad (30)$$

Substituting k_1, k_2, and k_3 into Equation (18) yields

$$P_1 = P_2 + k_1\ k_3 + \frac{k_2}{k_3} \qquad (31)$$

Equation (31) can be differentiated with respect to T_1 to give

$$\frac{dP_1}{dT_1} = k_1 \frac{dk_3}{dT_1} - \frac{k_2}{k_3}\frac{dk_3}{dT_1} \qquad (32)$$

By substituting Equation (30) into Equation (32) and rearranging, one obtains

$$\frac{dP_1}{dT_1} = \frac{\dfrac{-\left(k_1 - \dfrac{k_2}{k_3}\right) P_1}{T_1 Z_1} \left(\dfrac{1}{T_1} + \dfrac{1}{Z_1}\left(\dfrac{\partial Z_1}{\partial T_1}\right)_{P_1}\right)}{1 - \left(\dfrac{1}{T_1 Z_1}\right)\left(k_1 - \dfrac{k_2}{k_3}\right)} \tag{33}$$

Substituting the value of k_1, k_2, and k_3 into Equation (33) yields

$$\frac{dP_1}{dT_1} = \frac{-\left(9.374 \times 10^{-3}\ GL - \dfrac{24.99\ Q_0^2\ GLf}{d^5\left(\dfrac{P_1}{T_1 Z_1} + \dfrac{P_2}{T_2 Z_2}\right)}\right)\left(\dfrac{P_1}{T_1 Z_1}\right)\left(\dfrac{1}{T_1} + \dfrac{1}{Z_1}\left(\dfrac{\partial Z_1}{\partial T_1}\right)_{P_1}\right)}{1 - \left(\dfrac{1}{T_1 Z_1}\right)\left(9.374 \times 10^{-3}\ GL - \dfrac{24.99\ Q_0^2\ GLf}{d^5\left(\dfrac{P_1}{T_1 Z_1} + \dfrac{P_2}{T_2 Z_2}\right)}\right)}$$

. . . . .(34)

Similarly,

$$\frac{dP_1}{dT_2} = \frac{-\left(9.374 \times 10^{-3}\ GL - \dfrac{24.99\ Q_0^2\ GLf}{d^5\left(\dfrac{P_1}{T_1 Z_1} + \dfrac{P_2}{T_2 Z_2}\right)}\right)\left(\dfrac{P_2}{T_2 Z_2}\right)\left(\dfrac{1}{T_2} + \dfrac{1}{Z_2}\left(\dfrac{\partial Z_2}{\partial T_2}\right)_{P_2}\right)}{1 - \left(\dfrac{1}{T_2 Z_2}\right)\left(9.374 \times 10^{-3}\ GL - \dfrac{24.99\ Q_0^2\ GLf}{d^5\left(\dfrac{P_1}{T_1 Z_1} + \dfrac{P_2}{T_2 Z_2}\right)}\right)}$$

. . . . .(35)

5. Effect of gas gravity, G.

Let $k_1 = 9.374 \times 10^{-3}\ L$

$$k_2 = \frac{24.99\ Q_o^2\ Lf}{d^5}$$

$$k_3 = \frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}$$

Substituting k_1, k_2, and k_3 into Equation (18) yields

$$P_1 = P_2 + k_1\ k_3\ G + \frac{k_2}{k_3}\ G \tag{36}$$

Equation (36) can be differentiated with respect to G to give

$$\frac{dP_1}{dG} = k_1\ k_3 + \frac{k_1 G}{T_1 Z_1}\ \frac{dP_1}{dG} + \frac{k_2}{k_3} - \frac{k_2\ G}{T_1 Z_1\ k_3}\ \frac{dP_1}{dG} \tag{37}$$

Substituting the value of k_1, k_2, and k_3 into Equation (37) and rearranging yields

$$\frac{dP_1}{dG} = \frac{9.374 \times 10^{-3}\ L \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}\right) + \frac{24.99\ Q_o^2\ Lf}{d^5 \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}\right)}}{1 + \frac{G}{T_1 Z_1}\left(\frac{24.99\ Q_o\ Lf}{d^5 \left(\frac{P_1}{T_1 Z_1} + \frac{P_2}{T_2 Z_2}\right)} - 9.374 \times 10^{-3}\ L\right)}$$

. . . . .(38)

APPENDIX D

COPIES OF COMPUTER PROGRAMS

MAIN PROGRAM

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C       ****************************************************************
C       * SENSITIVITY ANALYSIS OF AVERAGE TEMPERATURE AND  *
C       * COMPRESSIBILITY FACTOR METHOD.                    *
C       *   THIS PROGRAM CALCULATES THE EFFECT OF           *
C       *   VARIATIONS IN THE INPUT PARAMETERS ON           *
C       *   THE CALCULATED BOTTOM-HOLE PRESSURE.            *
C       *     X(K)=1 VARIATION IN DEPTH                     *
C       *     X(K)=2 VARIATION IN FLOW RATE                 *
C       *     X(K)=3 VARIATION IN WELLHEAD TUBING PRESSURE *
C       *     X(K)=4 VARIATION IN BOTTOM-HOLE TEMPERATURE  *
C       *     X(K)=5 VARIATION IN WELLHEAD TEMPERATURE     *
C       ****************************************************************
C       NOMENCLATURE
C
C       YMOLE = MOLE FRACTION
C       YMW   = MOLECULAR WEIGHT
C       TEMC  = CRITICAL TEMPERATURE,DEGREE R
C       PREC  = CRITICAL PRESSURE,PSIA
C       DIAM  = INSIDE DIAMETER OF FLOW PIPE,INCHES
C       TSD   = WELLHEAD TEMPERATURE,DEGREE F
C       TDD   = BOTTOM-HOLE TEMPERATURE,DEGREE F
C       PSD   = WELLHEAD PRESSURE,PSIA
C       Q     = FLOW RATE
C       TRC   = REDUCED TEMPERATURE
C       PRC   = REDUCED PRESSURE
C       ZTEMC = PSEUDOCRITICAL TEMPERATURE,DEGREE R
C       ZPREC = PSEUDOCRITICAL PRESSURE,PSIA
C       SG    = GAS GRAVITY
C       DENS  = DENSITY
C       REN   = REYNOLDS NUMBER
C       AVEMW = AVERAGE MOLECULAR WEIGHT
C       VISM  = VISCOSITY,CP
C       TEMC  = CRITICAL TEMPERATURE,DEGREE R
C       PREC  = CRITICAL PRESSURE,PSIA
C

        DIMENSION YMOLE(16),YMW(16),TEMC(16),PREC(16),A(8)
       *,X(5)
        DATA A/0.31506237,1.0467099,0.57832729,0.53530771,
       1 0.61232032,0.10488813,0.68157001,0.68446549/
        E=0.0006

C       READ AND WRITE INPUT DATA                          .
C

        WRITE(6,208)
  208   FORMAT('1')
        READ(5,201) (YMW(I),TEMC(I),PREC(I),I=1,16)
  201   FORMAT(3F10.5)
```

```
      WRITE(6,212)
  212 FORMAT(/////,29X,'MOL. WT.',10X,'CRIT. TEMP.',8X
     *,'CRIT. PRESS.',/,
     147X,'DEG. RANKIN',12X,'PSIA',/)
      WRITE(6,202)(YMW(I),TEMC(I),PREC(I),I=1,16)
  202 FORMAT(10X,'CO2      ',3F20.5,/,10X,'N2       ',3F20.5,/
     *,10X,'H2S      '
     1,3F20.5,/,10X,'C1       ',3F20.5,/,10X,'C2       ',3F20.5,
     */,10X,'C3
     1  ',3F20.5,/,10X,'I-C4     ',3F20.5,/,10X,'N-C4     '
     *,3F20.5,/,10X,'I-
     1C5    ',3F20.5,/,10X,'N-C5     ',3F20.5,/,10X,'I-C6     '
     *,3F20.5,/,10X,
     1'N-C6     ',3F20.5,/,10X,'I-C7     ',3F20.5,/,10X,'N-C7
     * ',3F20.5,/,
     110X,'N-C8     ',3F20.5,/,10X,'HE       ',3F20.5)
      READ(5,206) (YMOLF(I),I=1,16)
  206 FORMAT(5F15.8)
      WRITE(6,215)
  215 FORMAT('1',////////,27X,'MOLE FRACTION',/)

C      COMPOSITION OF COMPONENTS MUST BE ENTERED IN
C      THE SAME ORDER AS THE OUTPUT FORMAT 202
C

      WRITE(6,207)(YMOLF(I),I=1,16)
  207 FORMAT (10X,'CO2      ',F20.5,/,10X,'N2       ',F20.5,/
     *,10X,'H2S      ',
     1F20.5,/,10X,'C1       ',F20.5,/,10X,'C2       ',F20.5,/
     *,10X,'C3       ',
     1F20.5,/,10X,'I-C4     ',F20.5,/,10X,'N-C4     ',F20.5,/
     *,10X,'I-C5     ',
     1F20.5,/,10X,'N-C5     ',F20.5,/,10X,'I-C6     ',F20.5,/
     *,10X,'N-C6     ',
     1F20.5,/,10X,'I-C7     ',F20.5,/,10X,'N-C7     ',F20.5,/
     *,10X,'N-C8     ',
     1F20.5,/,10X,'HE       ',F20.5,/////)
      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      WRITE(6,219)
  219 FORMAT(///,10X,'PRODUCTION PROBLEM')
      WRITE(6,216) DIAM,DEPTH
      WRITE(6,217) Q,TSOD,TSD,TDD,PSD
  216 FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,'
     * INCHES',/,
     110X,'THE LENGTH OF FLOW STRING                    =',F10.3,'
     *  FEET')
  217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,'  MSCF/DAY',/,
     110X,'THE AVE. TEMP. OF THE GROUND SURFACE   =',F10.3,'
     *  F',/,
```

```
     110X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',F10.3,'
     *  F',/,
     110X,'THE TEMP. OF THE PRODUCING FORMATION   =',F10.3,'
     *  F',/,
     110X,'THE WELL HEAD TUBING PRESSURE          =',F10.3,'
     *  PSIA',///)

C     WRITE OUTPUT HEADING
C

      WRITE (6,111)
  111 FORMAT ('1',///,20X,'SENSITIVITY ANALYSIS OF AVE T AND
     * Z METHOD')
      WRITE (6,112)
  112 FORMAT (//,18X,'DEPTH',12X,'Q',12X,'P2',13X,'T1',13X
     *,'T2',4X,
     1'BOTTOM HOLE PRESSURE',2X,' CHANGE',//)
      X(1)=DEPTH
      X(2)=Q
      X(3)=PSD
      X(4)=TDD
      X(5)=TSD
      DO 11 K=1,5
      ORIGN=X(K)
      DO 10 J=1,22
      X(K)=(1.+(J-11)/100.)*ORIGN
      IF (J-21) 2,2,1
    1 X(K)=ORIGN
      GO TO 11
    2 DEPTH=X(1)
      Q=X(2)
      PSD=X(3)
      TDD=X(4)
      TSD=X(5)

C     BOTTOM HOLE PRESSURE CALCULATION USING AVERAGE TEMP
C     AND AVERAGE COMPRESSIBILITY FACTOR METHOD
C

      TDD=TDD+459.67
      TSD=TSD+459.67
      TEM=(TDD+TSD)/2.

C     TRIAL AND ERROR FOR BOTTOM HOLE PRESSURE
C     ASSUME BOTTOM HOLE PRESSURE EQUALS WELLHEAD PRESSURE
C

      PRE1=PSD
  555 PRE=PRE1
      PRE=(PRE+PSD)/2.0
```

```
C     CALL SUBROUTINE PROP TO CACULATE THE
C     PSEUDOCRITICAL TEMPERATURE AND PRESSURE OF GAS
C
      CALL PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC,SG)
      TRC=TEM/ZTEMC
      PRC=PRE/ZPREC
C     CALL SUBROUTINE ZKATZ TO CACULATE Z-FACTOR
C
      CALL ZKATZ(TRC,PRC,Z,A)
      DENS=0.00149256*AVEMW*PRE/(Z*TEM)
C     CALL SUBROUTINE VISCO TO CACULATE THE GAS VISCOSITY
C
      CALL VISCO(TEM,PRE,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
C     CALL SUBROUTINE FRICT TO CACULATE FRICTION FACTOR
C
      CALL FRICT (E,DIAM,FF,REN)
      S=2.0*SG*DEPTH/(53.34*TEM*Z)
      T=(PSD**2)*(EXP(S))
      W=25.0*((Q/1000.0)**2)*SG*TEM*FF*DEPTH*Z
      V=EXP(S)-1.0
      U=(DIAM**5)*S
      PP=T+(W*V)/U
      PRE2=PP**0.5
      DELP=ABS(PRE1-PRE2)
C     COMPARE CALCULATED P1 WITH ASSUMED P1
C     IF NOT EQUAL,ASSUME P1 = CALCULATED P1
C
      IF (DELP-0.1) 556,556,560
  560 PRE1=PRE2
      GO TO 555
  556 DIF=3721.77966
      ERR=((PRE1-DIF)/DIF)*100.
      WRITE (6,113) DEPTH,Q,PSD,TDD,TSD,PRE1,ERR
  113 FORMAT (10X,7F15.5)
   10 CONTINUE
   11 CONTINUE
      CALL EXIT
      END
```

MAIN PROGRAM

```
C     ***********************************************************
C     * SENSITIVITY ANALYSIS OF AVERAGE DENSITY METHOD          *
C     *    THIS PROGRAM CALCULATES THE EFFECT OF                *
C     *    VARIATIONS IN THE INPUT PARAMETERS ON                *
C     *    THE CALCULATED BOTTOM-HOLE PRESSURE.                 *
C     *       X(K)=1 VARIATION IN DEPTH                         *
C     *       X(K)=2 VARIATION IN FLOW RATE                     *
C     *       X(K)=3 VARIATION IN WELLHEAD TUBING PRESSURE      *
C     *       X(K)=4 VARIATION IN BOTTOM-HOLE TEMPERATURE       *
C     *       X(K)=5 VARIATION IN WELLHEAD TEMPERATURE          *
C     ***********************************************************
C     NOMENCLATURE
C
C     YMOLF  = MOLE FRACTION
C     YMW    = MOLECULAR WEIGHT
C     TEMC   = CRITICAL TEMPERATURE,DEGREE R
C     PREC   = CRITICAL PRESSURE,PSIA
C     DIAM   = INSIDE DIAMETER OF FLOW PIPE,INCHES
C     TSD    = WELLHEAD TEMPERATURE,DEGREE F
C     TDD    = BOTTOM-HOLE TEMPERATURE,DEGREE F
C     PSD    = WELLHEAD PRESSURE,PSIA
C     Q      = FLOW RATE
C     TRC    = REDUCED TEMPERATURE
C     PRC    = REDUCED PRESSURE
C     ZTEMC  = PSEUDOCRITICAL TEMPERATURE,DEGREE R
C     ZPREC  = PSEUDOCRITICAL PRESSURE,PSIA
C     SG     = GAS GRAVITY
C     DENS   = DENSITY
C     REN    = REYNOLDS NUMBER
C     AVEMW  = AVERAGE MOLECULAR WEIGHT
C     VISM   = VISCOSITY,CP
C     TEMC   = CRITICAL TEMPERATURE,DEGREE R
C     PREC   = CRITICAL PRESSURE,PSIA
C

      DIMENSION YMOLF(16),YMW(16),TEMC(16),PREC(16),A(8)
     *,X(5)
      DATA A/0.31506237,1.0467099,0.57832729,0.53530771,
     1 0.61232032,0.10488813,0.68157001,0.68446549/

C     READ AND WRITE INPUT DATA
C

      WRITE(6,208)
  208 FORMAT('1')
      READ(5,201) (YMW(I),TEMC(I),PREC(I),I=1,16)
  201 FORMAT(3F10.5)
      WRITE(6,212)
  212 FORMAT(/////,29X,'MOL. WT.',10X,'CRIT. TEMP.',8X
```

```
     *,'CRIT. PRESS.',/,
     147X,'DEG. RANKIN',12X,'PSIA',/)
      WRITE(6,202)(YMW(I),TEMC(I),PREC(I),I=1,16)
  202 FORMAT(10X,'CO2     ',3F20.5,/,10X,'N2      ',3F20.5,/
     *,10X,'H2S     '
     1,3F20.5,/,10X,'C1      ',3F20.5,/,10X,'C2      ',3F20.5,
     */,10X,'C3
     1  ',3F20.5,/,10X,'I-C4    ',3F20.5,/,10X,'N-C4    '
     *,3F20.5,/,10X,'I-
     1C5    ',3F20.5,/,10X,'N-C5    ',3F20.5,/,10X,'I-C6    '
     *,3F20.5,/,10X,
     1'N-C6    ',3F20.5,/,10X,'I-C7    ',3F20.5,/,10X,'N-C7
     * ',3F20.5,/,
     110X,'N-C8    ',3F20.5,/,10X,'HE      ',3F20.5)
      READ(5,206) (YMOLF(I),I=1,16)
  206 FORMAT(5F15.8)
      WRITE(6,215)
  215 FORMAT('1',////////,27X,'MOLE FRACTION',/)
C
C     COMPOSITION OF COMPONENTS MUST BE ENTERED IN
C     THE SAME ORDER AS THE OUTPUT FORMAT 202
C

      WRITE(6,207)(YMOLF(I),I=1,16)
  207 FORMAT (10X,'CO2     ',F20.5,/,10X,'N2      ',F20.5,/
     *,10X,'H2S     ',
     1F20.5,/,10X,'C1      ',F20.5,/,10X,'C2      ',F20.5,/
     *,10X,'C3      ',
     1F20.5,/,10X,'I-C4    ',F20.5,/,10X,'N-C4    ',F20.5,/
     *,10X,'I-C5    ',
     1F20.5,/,10X,'N-C5    ',F20.5,/,10X,'I-C6    ',F20.5,/
     *,10X,'N-C6    ',
     1F20.5,/,10X,'I-C7    ',F20.5,/,10X,'N-C7    ',F20.5,/
     *,10X,'N-C8    ',
     1F20.5,/,10X,'HE      ',F20.5,/////)
      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      WRITE(6,219)
  219 FORMAT(///,10X,'PRODUCTION PROBLEM')
      WRITE(6,216) DIAM,DEPTH
      WRITE(6,217) Q,TSOD,TSD,TDD,PSD
  216 FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,'
     * INCHES',/,
     110X,'THE LENGTH OF FLOW STRING                =',F10.3,'
     *  FEET')
  217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,'  MSCF/DAY',/,
     110X,'THE AVE. TEMP. OF THE GROUND SURFACE    =',F10.3,'
     *  F',/,
     110X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',F10.3,'
     *  F',/,
```

```
     110X,'THE TEMP. OF THE PRODUCING FORMATION   =',F10.3,'
     *  F',/,
     110X,'THE WELL HEAD TUBING PRESSURE          =',F10.3,'
     *  PSIA',///)
C      WRITE OUTPUT HEADING
C
       WRITE (6,111)
  111  FORMAT ('1',///,20X,'SENSITIVITY ANALYSIS OF AVERAGE
     *DENSITY METHOD')
       WRITE (6,112)
  112  FORMAT (//,18X,'DEPTH',12X,'Q',12X,'P2',13X,'T1',13X
     *,'T2',4X,
     1'BOTTOM HOLE PRESSURE',2X,' CHANGE',//)
       X(1)=DEPTH
       X(2)=Q
       X(3)=PSD
       X(4)=TDD
       X(5)=TSD
       DO 11 K=1,5
       ORIGN=X(K)
       DO 10 J=1,22
       X(K)=(1.+(J-11)/100.)*ORIGN
       IF (J-21) 2,2,1
    1  X(K)=ORIGN
       GO TO 11
    2  DEPTH=X(1)
       Q=X(2)
       PSD=X(3)
       TDD=X(4)
       TSD=X(5)
C      BOTTOM-HOLE PRESSURE CALCULATION USING AVERAGE
C      DENSITY METHOD
C
       E=0.0006
       TDD=TDD+459.67
       TSD=TSD+459.67
C      TRIAL AND ERROR FOR BOTTOM HOLE PRESSURE
C      ASSUME BOTTOM HOLE PRESSURE EQUALS WELLHEAD PRESSURE
C
       PRE1=PSD
C      CALL SUBROUTINE PROP TO CACULATE THE
C      PSEUDOCRITICAL TEMPERATURE AND PRESSURE OF GAS
C
```

```
       MAIN  PROGRAM              ...(CONT'D)

      CALL PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC,SG)
  555 TRC1=TDD/ZTEMC
      PRC1=PRE1/ZPREC
C     CALL SUBROUTINE ZKATZ TO CACULATE Z-FACTOR
C

      CALL ZKATZ(TRC1,PRC1,Z1,A)
      TRC2=TSD/ZTEMC
      PRE2=PSD
      PRC2=PRE2/ZPREC
      CALL ZKATZ(TRC2,PRC2,Z2,A)
C     CALCULATE AVERAGE DENSITY
C

      DENS1=0.00149256*AVEMW*PRE1/(Z1*TDD)
      DENS2=0.00149256*AVEMW*PRE2/(Z2*TSD)
      DENS=(DENS1+DENS2)/2.0
      TEM=(TDD+TSD)/2.
      PRE=(PRE1+PRE2)/2.0
C     CALL SUBROUTINE VISCO TO CACULATE THE GAS VISCOSITY
C

      CALL VISCO(TEM,PRE,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
C     CALL SUBROUTINE FRICT TO CACULATE FRICTION FACTOR
C

      CALL FRICT (E,DIAM,FF,REN)
      S=0.009374*SG*DEPTH
      T=PRE1/(TDD*Z1)+PSD/(TSD*Z2)
      U=DIAM**5.0
      V=24.99*((Q/1000.0)**2.0)*DEPTH*SG*FF
      PRE3=PSD+(S*T)+V/(U*T)
      DELP=ABS(PRE3-PRE1)
C     COMPARE CALCULATED P1 WITH ASSUMED P1
C     IF NOT EQUAL,ASSUME P1 = CALCULATED P1
C

      IF (DELP-0.1) 556,556,560
  560 PRE1=PRE3
      GO TO 555
  556 DIF=3714.31245
      ERR=((PRE1-DIF)/DIF)*100.
      WRITE (6,113) DEPTH,Q,PSD,TDD,TSD,PRE1,ERR
  113 FORMAT (10X,7F15.5)
```

```
        MAIN  PROGRAM               ...(CONT'D)

   10 CONTINUE
   11 CONTINUE
      CALL EXIT
      END
```

MAIN PROGRAM

```
C     ****************************************************
C     * SENSITIVITY ANALYSIS OF CULLENDER AND SMITH      *
C     * METHOD                                           *
C     *    THIS PROGRAM CALCULATES THE EFFECT OF         *
C     *    VARIATIONS IN THE INPUT PARAMETERS ON         *
C     *    THE CALCULATED BOTTOM-HOLE PRESSURE.          *
C     *       X(K)=1 VARIATION IN DEPTH                  *
C     *       X(K)=2 VARIATION IN FLOW RATE              *
C     *       X(K)=3 VARIATION IN WELLHEAD TUBING PRESSURE *
C     *       X(K)=4 VARIATION IN BOTTOM-HOLE TEMPERATURE  *
C     *       X(K)=5 VARIATION IN WELLHEAD TEMPERATURE   *
C     ****************************************************
C     NOMENCLATURE
C
C     YMOLF = MOLE FRACTION
C     YMW   = MOLECULAR WEIGHT
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C     DIAM  = INSIDE DIAMETER OF FLOW PIPE,INCHES
C     TSD   = WELLHEAD TEMPERATURE,DEGREE F
C     TDD   = BOTTOM-HOLE TEMPERATURE,DEGREE F
C     PSD   = WELLHEAD PRESSURE,PSIA
C     Q     = FLOW RATE
C     TRC   = REDUCED TEMPERATURE
C     PRC   = REDUCED PRESSURE
C     ZTEMC = PSEUDOCRITICAL TEMPERATURE,DEGREE R
C     ZPREC = PSEUDOCRITICAL PRESSURE,PSIA
C     SG    = GAS GRAVITY
C     DENS  = DENSITY
C     REN   = REYNOLDS NUMBER
C     AVEMW = AVERAGE MOLECULAR WEIGHT
C     VISM  = VISCOSITY,CP
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C

      DIMENSION YMOLF(16),YMW(16),TEMC(16),PREC(16),A(8)
     *,X(5)
      DATA A/0.31506237,1.0467099,0.57832729,0.53530771,
     1 0.61232032,0.10488813,0.68157001,0.68446549/

C     READ AND WRITE INPUT DATA
C

      WRITE(6,208)
  208 FORMAT('1')
      READ(5,201) (YMW(I),TEMC(I),PREC(I),I=1,16)
  201 FORMAT(3F10.5)
      WRITE(6,212)
```

```
  212 FORMAT(/////,29X,'MOL. WT.',10X,'CRIT. TEMP.',8X
     *,'CRIT. PRESS.',/,
     147X,'DEG. RANKIN',12X,'PSIA',/)
      WRITE(6,202)(YMW(I),TEMC(I),PREC(I),I=1,16)
  202 FORMAT(10X,'CO2       ',3F20.5,/,10X,'N2        ',3F20.5,/
     *,10X,'H2S       '
     1,3F20.5,/,10X,'C1        ',3F20.5,/,10X,'C2        ',3F20.5,
     */,10X,'C3
     1  ',3F20.5,/,10X,'I-C4      ',3F20.5,/,10X,'N-C4      '
     *,3F20.5,/,10X,'I-
     1C5      ',3F20.5,/,10X,'N-C5      ',3F20.5,/,10X,'I-C6      '
     *,3F20.5,/,10X,
     1'N-C6      ',3F20.5,/,10X,'I-C7      ',3F20.5,/,10X,'N-C7
     * ',3F20.5,/,
     110X,'N-C8      ',3F20.5,/,10X,'HE        ',3F20.5)
      READ(5,206) (YMOLF(I),I=1,16)
  206 FORMAT(5F15.8)
      WRITE(6,215)
  215 FORMAT('1',////////,27X,'MOLE FRACTION',/)

C     COMPOSITION OF COMPONENTS MUST BE ENTERED IN
C     THE SAME ORDER AS THE OUTPUT FORMAT 202
C

      WRITE(6,207)(YMOLF(I),I=1,16)
  207 FORMAT (10X,'CO2       ',F20.5,/,10X,'N2        ',F20.5,/
     *,10X,'H2S       ',
     1F20.5,/,10X,'C1        ',F20.5,/,10X,'C2        ',F20.5,/
     *,10X,'C3        ',
     1F20.5,/,10X,'I-C4      ',F20.5,/,10X,'N-C4      ',F20.5,/
     *,10X,'I-C5      ',
     1F20.5,/,10X,'N-C5      ',F20.5,/,10X,'I-C6      ',F20.5,/
     *,10X,'N-C6      ',
     1F20.5,/,10X,'I-C7      ',F20.5,/,10X,'N-C7      ',F20.5,/
     *,10X,'N-C8      ',
     1F20.5,/,10X,'HE        ',F20.5,/////)
      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      WRITE(6,219)
  219 FORMAT(///,10X,'PRODUCTION PROBLEM')
      WRITE(6,216) DIAM,DEPTH
      WRITE(6,217) Q,TSOD,TSD,TDD,PSD
  216 FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,'
     * INCHES',/,
     110X,'THE LENGTH OF FLOW STRING                 =',F10.3,'
     *  FEET')
  217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,'  MSCF/DAY',/,
     110X,'THE AVE. TEMP. OF THE GROUND SURFACE    =',F10.3,'
     *  F',/,
     110X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',F10.3,'
```

```
     *  F',/,
     110X,'THE TEMP. OF THE PRODUCING FORMATION   =',F10.3,'
     *  F',/,
     110X,'THE WELL HEAD TUBING PRESSURE          =',F10.3,'
     *  PSIA',///)

C     WRITE OUTPUT HEADING
C

      WRITE (6,111)
  111 FORMAT ('1',///,30X,'SENSITIVITY ANALYSIS OF CULLENDER
     *AND SMITH MEYHOD')
      WRITE (6,112)
  112 FORMAT (//,18X,'DEPTH',12X,'Q',12X,'P2',13X,'T1',13X
     *,'T2',4X,
     1'BOTTOM HOLE PRESSURE',2X,' CHANGE',//)
      X(1)=DEPTH
      X(2)=Q
      X(3)=PSD
      X(4)=TDD
      X(5)=TSD
      DO 11 K=1,5
      ORIGN=X(K)
      DO 10 J=1,22
      X(K)=(1.+(J-11)/100.)*ORIGN
      IF (J-21) 2,2,1
    1 X(K)=ORIGN
      GO TO 11
    2 DEPTH=X(1)
      Q=X(2)
      PSD=X(3)
      TDD=X(4)
      TSD=X(5)

C     BOTTOM HOLE PRESSURE CALCULATION USING CULLENDER AND
C     SMITH METHOD
C     USING A TWO STEP CALCULATION SCHEME
C

      GN1=0.0
      DEPTH=0.0
      TEM1=TSD+459.67
      PRE1=PSD
      SG=0.64

C     CALL SUBROUTINE PROP TO CACULATE THE
C     PSEUDOCRITICAL TEMPERATURE AND PRESSURE OF GAS
C

      CALL PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC,SG)
```

```
          MAIN  PROGRAM              ...(CONT'D)

      TRC=TEM1/ZTEMC
      PRC=PRE1/ZPREC

C     CALL SUBROUTINE ZKATZ TO CACULATE Z-FACTOR
C

      CALL ZKATZ(TRC,PRC,Z,A)
      D=PRE1/(Z*TEM1)
      B=D**2.0
      TEM=TEM1
      PRE=PRE1
      DENS=0.00149256*AVEMW*PRE/(Z*TEM)

C     CALL SUBROUTINE VISCO TO CACULATE THE GAS VISCOSITY
C

      CALL VISCO(TEM,PRE,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
      E=0.0006

C     CALL SUBROUTINE FRICT TO CACULATE FRICTION FACTOR
C

      CALL FRICT (E,DIAM,FF,REN)
      C=(666.6*((Q/1000.0)**2.0)*FF)/(DIAM**5.0)
      GN=D/(B+C)
      GN2=GN

C     CALCULATE I AT WELLHEAD CONDITIONS
C

      M=2.0*SG*DEPTH/53.34
      W=GN+GN1
      DELP=M/W
      PREN=PSD+DELP
      GN1=GN2
      DEPTH=X(1)/2.
      M=2.0*SG*DEPTH/53.34
      W=GN+GN1
      DELP=M/W
      PREN=PSD+DELP
      DIFF=ABS(PREN-PRE1)
      IF (DIFF-0.1) 560,560,555

C     CALCULATE I AT HALF-DEPTH
C

  555 PRE1=PREN
      TEM1=459.67+(TDD+TSD)/2.
      DEPTH=X(1)/2.
```

MAIN PROGRAM ...(CONT'D)

```
      PRC=PRE1/ZPREC
      TRC=TEM1/ZTEMC
      CALL ZKATZ(TRC,PRC,Z,A)
      D=PRE1/(Z*TEM1)
      B=D**2.0
      PRE=(PRE1+PSD)/2.0
      TEM=TEM1
      DENS=0.00149256*AVEMW*PRE/(Z*TEM)
      CALL VISCO(TEM,PRE,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
      CALL FRICT (E,DIAM,FF,REN)
      C=(666.6*((Q/1000.0)**2.0)*FF)/(DIAM**5.0)
      GN=D/(B+C)
      M=2.0*SG*DEPTH/53.34
      W=GN+GN2
      DELP=M/W
      PREN=PSD+DELP
      DIFF=ABS(PREN-PRE1)
      IF (DIFF-0.1) 560,560,555
C     CALCULATE I AT BOTTOM-HOLE CONDITIONS
C
  560 DEPTH=X(1)
      PRE3=PREN
      GN3=GN
      M=2.0*SG*DEPTH/(2.0*53.34)
      W=GN+GN3
      DELP=M/W
      PREN=PRE3+DELP
      DIFF=ABS(PREN-PRE1)
      IF (DIFF-0.1) 570,570,554
  554 DEPTH=X(1)
      TEM1=TDD+459.67
      PRE1=PREN
      TRC=TEM1/ZTEMC
      PRC=PRE1/ZPREC
      CALL ZKATZ(TRC,PRC,Z,A)
      D=PRE1/(Z*TEM1)
      B=D**2.0
      TEM=TEM1
      PRE=(PRE1+PSD)/2.0
      DENS=0.00149256*AVEMW*PRE/(Z*TEM)
      CALL VISCO(TEM,PRE,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
      CALL FRICT (E,DIAM,FF,REN)
      C=(666.6*((Q/1000.0)**2.0)*FF)/(DIAM**5.0)
      GN=D/(B+C)
      M=2.0*SG*DEPTH/(2.0*53.34)
      W=GN+GN3
```

```
              MAIN  PROGRAM              ...(CONT'D)

          DELP=M/W
          PREN=PRE3+DELP
          DIFF=ABS(PREN-PRE1)
          IF (DIFF-0.1) 570,570,554
      570 CONTINUE
C
C         USE SIMPSON'S RULE TO OBTAIN THE
C         BOTTOM-HOLE PRESSURE
C

          GN4=GN
          DEPTH=X(1)
          G=GN2+(4.0*GN3)+GN4
          F=SG*DEPTH*3.0*2.0
          P=PSD+(F/(53.34*G))
          DIF=1091.71616
          ERR=((P-DIF)/DIF)*100.
          WRITE (6,114) DEPTH,Q,PSD,TDD,TSD,P,ERR
      114 FORMAT (10X,7F15.5)
       10 CONTINUE
       11 CONTINUE
          CALL EXIT
          END
```

MAIN PROGRAM

```
C     ***************************************************************
C     * SENSITIVITY ANALYSIS OF POETTMANN METHOD                     *
C     *   THIS PROGRAM CALCULATES THE EFFECT OF                      *
C     *   VARIATIONS IN THE INPUT PARAMETERS ON                      *
C     *   THE CALCULATED BOTTOM-HOLE PRESSURE.                       *
C     *     X(K)=1 VARIATION IN DEPTH                                *
C     *     X(K)=2 VARIATION IN FLOW RATE                            *
C     *     X(K)=3 VARIATION IN WELLHEAD TUBING PRESSURE *
C     *     X(K)=4 VARIATION IN BOTTOM-HOLE TEMPERATURE  *
C     *     X(K)=5 VARIATION IN WELLHEAD TEMPERATURE     *
C     ***************************************************************
C     NOMENCLATURE
C
C     YMOLF = MOLE FRACTION
C     YMW   = MOLECULAR WEIGHT
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C     DIAM  = INSIDE DIAMETER OF FLOW PIPE,INCHES
C     TSD   = WELLHEAD TEMPERATURE,DEGREE F
C     TDD   = BOTTOM-HOLE TEMPERATURE,DEGREE F
C     PSD   = WELLHEAD PRESSURE,PSIA
C     Q     = FLOW RATE
C     TRC   = REDUCED TEMPERATURE
C     PRC   = REDUCED PRESSURE
C     ZTEMC = PSEUDOCRITICAL TEMPERATURE,DEGREE R
C     ZPREC = PSEUDOCRITICAL PRESSURE,PSIA
C     SG    = GAS GRAVITY
C     DENS  = DENSITY
C     REN   = REYNOLDS NUMBER
C     AVEMW = AVERAGE MOLECULAR WEIGHT
C     VISM  = VISCOSITY,CP
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C

      DIMENSION YMOLF(16),YMW(16),TEMC(16),PREC(16),A(8)
     *,X(5)
      DATA A/0.31506237,1.0467099,0.57832729,0.53530771,
     1 0.61232032,0.10488813,0.68157001,0.68446549/

C     READ AND WRITE INPUT DATA
C

      WRITE(6,208)
  208 FORMAT('1')
      READ(5,201) (YMW(I),TEMC(I),PREC(I),I=1,16)
  201 FORMAT(3F10.5)
      WRITE(6,212)
  212 FORMAT(/////,29X,'MOL. WT.',10X,'CRIT. TEMP.',8X
```

```
          MAIN  PROGRAM              ...(CONT'D)

     *,'CRIT. PRESS.',/,
     147X,'DEG. RANKIN',12X,'PSIA',/)
      WRITE(6,202)(YMW(I),TEMC(I),PREC(I),I=1,16)
  202 FORMAT(10X,'CO2      ',3F20.5,/,10X,'N2       ',3F20.5,/
     *,10X,'H2S      '
     1,3F20.5,/,10X,'C1       ',3F20.5,/,10X,'C2       ',3F20.5,
     */,10X,'C3
     1   ',3F20.5,/,10X,'I-C4     ',3F20.5,/,10X,'N-C4     '
     *,3F20.5,/,10X,'I-
     1C5     ',3F20.5,/,10X,'N-C5     ',3F20.5,/,10X,'I-C6     '
     *,3F20.5,/,10X,
     1'N-C6     ',3F20.5,/,10X,'I-C7     ',3F20.5,/,10X,'N-C7
     * ',3F20.5,/,
     110X,'N-C8     ',3F20.5,/,10X,'HE       ',3F20.5)
      READ(5,206) (YMOLF(I),I=1,16)
  206 FORMAT(5F15.8)
      WRITE(6,215)
  215 FORMAT('1',////////,27X,'MOLE FRACTION',/)
C
C     COMPOSITION OF COMPONENTS MUST BE ENTERED IN
C     THE SAME ORDER AS THE OUTPUT FORMAT 202
C

      WRITE(6,207)(YMOLF(I),I=1,16)
  207 FORMAT (10X,'CO2      ',F20.5,/,10X,'N2       ',F20.5,/
     *,10X,'H2S      ',
     1F20.5,/,10X,'C1       ',F20.5,/,10X,'C2       ',F20.5,/
     *,10X,'C3       ',
     1F20.5,/,10X,'I-C4     ',F20.5,/,10X,'N-C4     ',F20.5,/
     *,10X,'I-C5     ',
     1F20.5,/,10X,'N-C5     ',F20.5,/,10X,'I-C6     ',F20.5,/
     *,10X,'N-C6     ',
     1F20.5,/,10X,'I-C7     ',F20.5,/,10X,'N-C7     ',F20.5,/
     *,10X,'N-C8     ',
     1F20.5,/,10X,'HE       ',F20.5,/////)
      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      WRITE(6,219)
  219 FORMAT(///,10X,'PRODUCTION PROBLEM')
      WRITE(6,216) DIAM,DEPTH
      WRITE(6,217) Q,TSOD,TSD,TDD,PSD
  216 FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,'
     * INCHES',/,
     110X,'THE LENGTH OF FLOW STRING                =',F10.3,'
     *  FEET')
  217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,'  MSCF/DAY',/,
     110X,'THE AVE. TEMP. OF THE GROUND SURFACE    =',F10.3,'
     *  F',/,
     110X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',F10.3,'
     *  F',/,
```

MAIN PROGRAM ...(CONT'D)

```
     110X,'THE TEMP. OF THE PRODUCING FORMATION   =',F10.3,'
     *  F',/,
     110X,'THE WELL HEAD TUBING PRESSURE         =',F10.3,'
     *  PSIA',///)

C     WRITE OUTPUT HEADING
C

      WRITE (6,119)
  119 FORMAT('1',///,20X,
     *'SENSITIVITY ANALYSIS OF POETTMANN METHOD')
      WRITE (6,112)
  112 FORMAT (//,18X,'DEPTH',12X,'Q',12X,'P2',13X,'T1',13X
     *,'T2',4X,
     1'BOTTOM HOLE PRESSURE',2X,' CHANGE',//)
      X(1)=DEPTH
      X(2)=Q
      X(3)=PSD
      X(4)=TDD
      X(5)=TSD
      DO 33 K=1,5
      ORIGN=X(K)
      DO 44 J=1,22
      X(K)=(1.+(J-11)/100.)*ORIGN
      IF (J-21) 77,77,88
   88 X(K)=ORIGN
      GO TO 33
   77 DEPTH=X(1)
      Q=X(2)
      PSD=X(3)
      TDD=X(4)
      TSD=X(5)

C     BOTTOM HOLE PRESSURE CALCULATION USING
C     POETTMANN METHOD
C

      F=0.0006
      TDD=TDD+459.67
      TSD=TSD+459.67
      TEM=(TDD+TSD)/2.
      PRE1=PSD

C     CALL SUBROUTINE PROP TO CACULATE THE
C     PSEUDOCRITICAL TEMPERATURE AND PRESSURE OF GAS
C

      CALL PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC,SG)
      TRC2=TEM/ZTEMC
      PRC2=PRE1/ZPREC
```

```
      H=0.05
      Y1=PR2/H
      N=(2*Y1)/2
      NDIM=N-3
      DO 10 I=1,NDIM
      PR(I)=0.2+H*(I-1)
C     CALL SUBROUTINE ZKATZ TO CACULATE Z-FACTOR
C
      CALL ZKATZ (TR2,PR(I),Z(I),A)
   10 V(I)=Z(I)/PR(I)
C     CALL SUBROUTINE QSF TO EVALUATE THE INTEGRAL
C     OF Z/PR NUMERICALLY
C     A STEP SIZE OF 0.05 IN PR IS USED IN THE
C     NUMERICAL INTEGRATION
C
      CALL QSF (H,V,AIN,NDIM)
      ER1=PR2-PR(NDIM)
C     ASSIGN AN ERROR TOLERANCE OF 0.001 IN PR
C
      IF (ABS(ER1-0.001)) 22,22,111
   22 C2=AIN(NDIM)
      GO TO 95
C     USE TRAPEZOIDAL RULE TO OBTAIN THE VALUE OF
C     THE INTEGRAL AT THE REQUIRED PR
C
  111 V1=V(NDIM)
      A1=AIN(NDIM)
      CALL ZKATZ (TR2,PR2,Z2,A)
      V2=Z2/PR2
      C2=A1+(V1+V2)*ER1/2.
C     TRIAL AND ERROR FOR THE BOTTOM-HOLE PRESSURE
C     GUESS AN INITIAL VALUE FOR THE BOTTOM-HOLE PRESSURE
C
   95 P2=3000.0
   96 AP=(P2+PREF1)/2.
      AT=TEM
      TRA=AT/ZTEMC
      PRA=AP/ZPREC
      CALL ZKATZ (TRA,PRA,ZA,A)
      DENS=0.00149256*AVEMW*AP/(ZA*AT)
```

```
C     CALL SUBROUTINE VISCO TO CACULATE THE GAS VISCOSITY
C
      CALL VISCO (AT,AP,AVEMW,DENS,VISM)
      REN=20.094595*Q*SG/(VISM*DIAM)
C     CALL SUBROUTINE FRICT TO CACULATE FRICTION FACTOR
C
      CALL FRICT(E,DIAM,FF,REN)
      W=0.234293*FF*((Q/1000.)**2)*(SG**2)
      U=DIAM**5
      PR1=P2/ZPREC
      TR1=TR2
      H=0.05
      Y2=PR1/H
      N=(2*Y2)/2
      NDIM=N-3
      DO 21 I=1,NDIM
      PR(I)=0.2+H*(I-1)
      CALL ZKATZ (TR1,PR(I),Z(I),A)
   21 V(I)=Z(I)/PR(I)
      CALL QSF (H,V,AIN,NDIM)
      ER2=PR1-PR(NDIM)
      IF(ABS(ER2-0.001)) 55,55,66
   55 C1=AIN(NDIM)
      GO TO 94
   66 V1=V(NDIM)
      A2=AIN(NDIM)
      CALL ZKATZ (TR1,PR1,Z1,A)
      V2=Z1/PR1
      C1=A2+(V1+V2)*ER2/2.
   94 CONTINUE
      XS=(53.34*AT/SG)*(C1-C2)
      T=W*(XS**2)/U
      DP=((DEPTH*T)/(XS-DEPTH))**0.5
      PN=PSD+DP
      DELP=ABS(PN-P2)
      IF (DELP-0.1) 98,98,99
   99 P2=PN
      GO TO 96
   98 DIF=3635.29
      ERR=((PN-DIF)/DIF)*100.
      WRITE (6,113) DEPTH,Q,PSD,TDD,TSD,PN,ERR
  113 FORMAT (10X,7F15.5)
   44 CONTINUE
   33 CONTINUE
      CALL EXIT
      END
```

MAIN PROGRAM

```
C     ***************************************************
C     * SENSITIVITY ANALYSIS OF SUKKAR AND CORNELL METHOD*
C     *   THIS PROGRAM CALCULATES THE EFFECT OF         *
C     *   VARIATIONS IN THE INPUT PARAMETERS ON         *
C     *   THE CALCULATED BOTTOM-HOLE PRESSURE.          *
C     *     X(K)=1 VARIATION IN DEPTH                   *
C     *     X(K)=2 VARIATION IN FLOW RATE               *
C     *     X(K)=3 VARIATION IN WELLHEAD TUBING PRESSURE *
C     *     X(K)=4 VARIATION IN BOTTOM-HOLE TEMPERATURE  *
C     *     X(K)=5 VARIATION IN WELLHEAD TEMPERATURE    *
C     ***************************************************
C     NOMENCLATURE
C
C     YMOLF = MOLE FRACTION
C     YMW   = MOLECULAR WEIGHT
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C     DIAM  = INSIDE DIAMETER OF FLOW PIPE,INCHES
C     TSD   = WELLHEAD TEMPERATURE,DEGREE F
C     TDD   = BOTTOM-HOLE TEMPERATURE,DEGREE F
C     PSD   = WELLHEAD PRESSURE,PSIA
C     Q     = FLOW RATE
C     TRC   = REDUCED TEMPERATURE
C     PRC   = REDUCED PRESSURE
C     ZTEMC = PSEUDOCRITICAL TEMPERATURE,DEGREE R
C     ZPREC = PSEUDOCRITICAL PRESSURE,PSIA
C     SG    = GAS GRAVITY
C     DENS  = DENSITY
C     REN   = REYNOLDS NUMBER
C     AVEMW = AVERAGE MOLECULAR WEIGHT
C     VISM  = VISCOSITY,CP
C     TEMC  = CRITICAL TEMPERATURE,DEGREE R
C     PREC  = CRITICAL PRESSURE,PSIA
C

      DIMENSION YMOLF(16),YMW(16),TEMC(16),PREC(16),A(8)
     *,X(5)
      DATA A/0.31506237,1.0467099,0.57832729,0.53530771,
     1 0.61232032,0.10488813,0.68157001,0.68446549/

C     READ AND WRITE INPUT DATA
C

      WRITE(6,208)
  208 FORMAT('1')
      READ(5,201) (YMW(I),TEMC(I),PREC(I),I=1,16)
  201 FORMAT(3F10.5)
      WRITE(6,212)
  212 FORMAT(/////,29X,'MOL. WT.',10X,'CRIT. TEMP.',8X
```

```
     *,'CRIT. PRESS.',/,
     147X,'DEG. RANKIN',12X,'PSIA',/)
      WRITE(6,202)(YMW(I),TEMC(I),PREC(I),I=1,16)
  202 FORMAT(10X,'CO2    ',3F20.5,/,10X,'N2     ',3F20.5,/
     *,10X,'H2S    '
     1,3F20.5,/,10X,'C1     ',3F20.5,/,10X,'C2     ',3F20.5,
     */,10X,'C3
     1  ',3F20.5,/,10X,'I-C4   ',3F20.5,/,10X,'N-C4   '
     *,3F20.5,/,10X,'I-
     1C5   ',3F20.5,/,10X,'N-C5   ',3F20.5,/,10X,'I-C6   '
     *,3F20.5,/,10X,
     1'N-C6   ',3F20.5,/,10X,'I-C7   ',3F20.5,/,10X,'N-C7
     * ',3F20.5,/,
     110X,'N-C8   ',3F20.5,/,10X,'HE     ',3F20.5)
      READ(5,206) (YMOLF(I),I=1,16)
  206 FORMAT(5F15.8)
      WRITE(6,215)
  215 FORMAT('1',////////,27X,'MOLE FRACTION',/)
C
C     COMPOSITION OF COMPONENTS MUST BE ENTERED IN
C     THE SAME ORDER AS THE OUTPUT FORMAT 202
C

      WRITE(6,207)(YMOLF(I),I=1,16)
  207 FORMAT (10X,'CO2    ',F20.5,/,10X,'N2     ',F20.5,/
     *,10X,'H2S    ',
     1F20.5,/,10X,'C1     ',F20.5,/,10X,'C2     ',F20.5,/
     *,10X,'C3     ',
     1F20.5,/,10X,'I-C4   ',F20.5,/,10X,'N-C4   ',F20.5,/
     *,10X,'I-C5   ',
     1F20.5,/,10X,'N-C5   ',F20.5,/,10X,'I-C6   ',F20.5,/
     *,10X,'N-C6   ',
     1F20.5,/,10X,'I-C7   ',F20.5,/,10X,'N-C7   ',F20.5,/
     *,10X,'N-C8   ',
     1F20.5,/,10X,'HE     ',F20.5,/////)
      READ(5,206) DIAM,DEPTH,TSOD
      READ(5,206) Q,TSD,PSD,TDD
      WRITE(6,219)
  219 FORMAT(///,10X,'PRODUCTION PROBLEM')
      WRITE(6,216) DIAM,DEPTH
      WRITE(6,217) Q,TSOD,TSD,TDD,PSD
  216 FORMAT(//////,10X,'TUBING I. D.',27X,'=',F10.3,'
     * INCHES',/,
     110X,'THE LENGTH OF FLOW STRING              =',F10.3,'
     *  FEET')
  217 FORMAT(10X,'FLOW RATE',30X,'=',F10.3,'  MSCF/DAY',/,
     110X,'THE AVE. TEMP. OF THE GROUND SURFACE   =',F10.3,'
     *  F',/,
     110X,'THE TEMP. OF FLOWING STREAM AT SURFACE =',F10.3,'
     *  F',/,
```

```
          MAIN  PROGRAM             ...(CONT'D)

     110X,'THE TEMP. OF THE PRODUCING FORMATION   =',F10.3,'
     *  F',/,
     110X,'THE WELL HEAD TUBING PRESSURE           =',F10.3,'
     *  PSIA',///)
C      WRITE OUTPUT HEADING
C
      WRITE (6,119)
  119 FORMAT('1',///,20X,
     *'SENSITIVITY ANALYSIS OF SUKKAR-CORNELL METHOD')
      WRITE (6,112)
  112 FORMAT (//,18X,'DEPTH',12X,'Q',12X,'P2',13X,'T1',13X
     *,'T2',4X,
     1'BOTTOM HOLE PRESSURE',2X,' CHANGE',//)
      X(1)=DEPTH
      X(2)=Q
      X(3)=PSD
      X(4)=TDD
      X(5)=TSD
      DO 33 K=1,5
      ORIGN=X(K)
      DO 44 J=1,22
      X(K)=(1.+(J-11)/100.)*ORIGN
      IF (J-21) 77,77,88
   88 X(K)=ORIGN
      GO TO 33
   77 DEPTH=X(1)
      Q=X(2)
      PSD=X(3)
      TDD=X(4)
      TSD=X(5)
C      BOTTOM-HOLE PRESSURE CALCULATION USING
C      SUKKAR AND CORNELL METHOD
C
      TDD=TDD+459.67
      TSD=TSD+459.67
      TEM=(TDD+TSD)/2.
      PRE1=PSD
      AT=TEM
C      CALL SUBROUTINE PROP TO CACULATE THE
C      PSEUDOCRITICAL TEMPERATURE AND PRESSURE OF GAS
C
      CALL PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC,SG)
      TRC2=TEM/ZTEMC
      PRC2=PRE1/ZPREC
```

```
            MAIN   PROGRAM             ...(CONT'D)

      EF=0.01482
      B=(666.6*EF*((Q/1000.)**2)*(AT**2))/((DIAM**5)*(ZPREC*
     **2))
      H=0.05
      Y1=PR2/H
      N=(2*Y1)/2
      NDIM=N-3
      DO 10 I=1,NDIM
      PR(I)=0.2+H*(I-1)
C     CALL SUBROUTINE ZKATZ TO CACULATE Z-FACTOR
C

      CALL ZKATZ (TR2,PR(I),Z(I),A)
   10 V(I)=(Z(I)/PR(I))/(1.+(B*(Z(I)/PR(I))**2))
C     CALL SUBROUTINE QSF TO EVALUATE THE INTEGRAL
C     OF (Z/PR)/(1.+(B*(Z/PR)**2)) NUMERICALLY
C     A STEP SIZE OF 0.05 IN PR IS USED IN THE
C     NUMERICAL INTEGRATION
C

      CALL QSF (H,V,AIN,NDIM)
      ER1=PR2-PR(NDIM)
C     ASSIGN AN ERROR TOLERANCE OF 0.001 IN PR
C

      IF (ABS(ER1-0.001)) 22,22,111
   22 C2=AIN(NDIM)
      GO TO 95
C     USE TRAPEZOIDAL RULE TO OBTAIN THE VALUE OF
C     THE INTEGRAL AT THE REQUIRED PR
C

  111 V1=V(NDIM)
      A1=AIN(NDIM)
      CALL ZKATZ (TR2,PR2,Z2,A)
      V2=Z2/PR2
      C2=A1+(V1+V2)*ER1/2.
   95 W=(SG*DEPTH)/(53.34*AT)
      AIN1=C2+W
      TR1=TEM/ZTEMC
C     AN ERROR TOLERANCE OF 0.00001 IS ASSIGNED
C     IN MATCHING THE CALCULATED INTEGRAL
C

      E=0.00001
```

```
          MAIN  PROGRAM            ...(CONT'D)

      NDIM=NDIM+NDIM
      DO 11 I=1,NDIM
      PR(I)=0.2+H*(I-1)
      CALL ZKATZ (TR1,PR(I),Z(I),A)
   11 V(I)=(Z(I)/PR(I))/(1.+(B*(Z(I)/PR(I))**2))
      CALL QSF (H,V,AIN,NDIM)
      GO TO 4
    3 NDIM=NDIM-1
C
C     USE THE METHOD OF INTERVAL HALVING TO OBTAIN THE
C     VALUE OF THE PR THAT GIVES AN INTEGRAL VALUE
C     EQUALS TO THE CALCULATED INTEGRAL VALUE AT THE
C     BOTTOM-HOLE CONDITIONS
C

    4 IF (AIN(NDIM)-AIN1) 1,1,2
    2 GO TO 3
    1 XH=AIN(NDIM+1)
      PH=PR(NDIM+1)
      XL=AIN(NDIM)
      PL=PR(NDIM)
   20 XM=(XH+XL)/2.
      PM=(PH+PL)/2.
      IF (ABS(XM-AIN1)-E) 5,5,14
   14 IF (XM-AIN1) 9,5,6
    6 XH=XM
      PH=PM
      GO TO 20
    9 XL=XM
      PL=PM
      GO TO 20
    5 P=PM
      PB=P*ZPREC
      DIF=1072.41729
      ERR=((PB-DIF)/DIF)*100.
      WRITE (6,113) DEPTH,0,PSD,TDD,TSD,PB,ERR
  113 FORMAT (10X,7F15.5)
   44 CONTINUE
   33 CONTINUE
      CALL EXIT
      END
```

SUBROUTINE PROP

```
      SUBROUTINE PROP(AVEMW,ZPREC,ZTEMC,YMOLF,YMW,TEMC,PREC
     *,SG)
C     THIS SUBROUTINE CALCULATES THE PSEUDOCRITICAL
C     TEMPERATURE AND PRESSURE OF THE GAS MIXTURE.
C     AZIZ AND WICHERT CORRECTION TO THE STANDING-
C     KATZ CHART IS USED TO CORRECT FOR THE PRESENCE
C     OF SOUR GASES
C
      DIMENSION YMOLF(16),YMW(16),TEMC(16),PREC(16)
      M=1
C     M TAKES ON THE VALUE OF 0 OR 1
C     IF M=0,PSEUDOCRITICAL PROPERTIES ARE CALCULATED
C     BY THE SPECIFIC GRAVITY CORRELATION BY BROWN AND
C     ETAL..SOUR GAS IS CORRECTED BY CARR'S METHOD
C     AND WICHERT AND AZIZ CORRECTION FACTOR
C
C     IF M=1,PSEUDOCRITICAL PROPERTIES ARE CALCULATED
C     USING MOLAL AVERAGE AND CORRECTED FOR SOUR GASES
C     USING AZIZ AND WICHERT CORRECTION TO STANDING-
C     KATZ CHART
C
      IF (M-1) 6,4,4
    4 AVEMW=0.0
      PTEMC=0.0
      PPREC=0.0
      DO 5 I=1,16
      AVEMW=AVEMW+YMOLF(I)*YMW(I)
      PTEMC=PTEMC+YMOLF(I)*TEMC(I)
    5 PPREC=PPREC+YMOLF(I)*PREC(I)
      SG=AVEMW/28.97
      GO TO 9
    6 PTEMC=171.137+313.725*SG
      PTEMC=PTEMC-80.0*YMOLF(1)-250.*YMOLF(2)+130.*YMOLF(3)
      IF (SG-0.85) 7,7,8
    7 PPREC=695.1-40.*SG
      GO TO 103
    8 PPREC=704.396-51.724*SG
  103 PPREC=PPREC+440.*YMOLF(1)-170.*YMOLF(2)+600.*YMOLF(3)
    9 S1=YMOLF(1)+YMOLF(3)
      S2=YMOLF(3)
      E3=120.0*(S1**0.90-S1**1.6)+15.0*(S2**0.50-S2**4)
      S4=PTEMC+S2*(1.0-S2)*E3
      ZTEMC=PTEMC-E3
      ZPREC=PPREC*ZTEMC/S4
      RETURN
      END
```

SUBROUTINE ZKATZ

```
      SUBROUTINE ZKATZ(TR,PR,Z,A)
C     THIS SUBROUTINE CALCULATES THE Z-FACTOR USING A
C     REDUCED B-W-R EQUATION. THE B-W-R COEFFICIENTS
C     WERE CALCULATED FROM THE TABULAR DATA IN
C     'HANDBOOK OF NATURAL GAS ENGINEERING'.
C

      DIMENSION A(8)
      TRC=TR
      PRC=PR
      ITER=0
      DR=1.0
      IF(TRC-1.05) 43,13,13
   13 IF(TRC-3.0) 14,14,43
   14 IF(PRC-30.0) 15,15,43
   15 DO 20 ITER=1,10
      DR2=DR*DR
      T1=(A(1)*TRC-A(2)-A(3)/TRC**2)*DR
      T2=(A(4)*TRC-A(5))*DR2
      T3=A(5)*A(6)*DR**5
      T4=A(7)*DR2/TRC**2
      T5=A(8)*DR2
      T6=EXP(-T5)
      P=(TRC+T1+T2+T3)*DR+T4*DR*(1.0+T5)*T6
      DP=TRC+2.0*T1+3.0*T2+6.0*T3+T4*T6*(3.0+3.0*T5-2.0*T5
     **T5)
      DR1=DR-(P-0.270*PRC)/DP
      IF(DR1) 16,16,17
   16 DR1=0.5*DR
   17 IF(DR1-2.2) 19,19,18
   18 DR1=DR+0.9*(2.2-DR)
   19 IF(ABS(DR-DR1)-0.00001) 21,20,20
   20 DR=DR1
   21 Z=0.270*PRC/(DR1*TRC)
   43 CONTINUE
      RETURN
      END
```

SUBROUTINE VISCO

```
      SUBROUTINE VISCO(TEM,PRE,AVEMW,DENS,VISM)
C     THIS SUBROUTINE CALCULATES THE VISCOSITY OF
C     THE GAS MIXTURE
C
      VK=(9.4+0.02*AVEMW)*TEM**1.5/(209.0+19.0*AVEMW+TEM)
      VX=3.5+986.0/TEM+0.01*AVEMW
      VY=2.4-0.2*VX
      VY1=VX*DENS**VY
      VISM=VK*EXP(VY1)/10000.0
      RETURN
      END
```

SUBROUTINE FRICT

```
      SUBROUTINE FRICT(E,DIAM,FF,REN)
C     THIS SUBROUTINE CALCULATES THE FRICTION FACTOR
C
      DIMENSION G(100)
      C=ALOG(10.0)
      RR=E/DIAM
      IF(RR) 28,28,27
   27 RENCR=3200.0/RR
      IF(REN-RENCR) 28,32,32
   28 X=0.08
      DO 30 I=1,100
      X1=RR/3.7+2.51/(REN*X)
      X2=ALOG(X1)/2.302585
      FX=1.0+2.0*X*X2
      FXD=2.0*X2-5.02/(REN*X*X1*C)
      G(I)=X-FX/FXD
      X=G(I)
      IF(I-1) 30,30,29
   29 IF(ABS(G(I)-G(I-1))-0.000001) 31,31,30
   30 CONTINUE
   31 FF=X*X
      GO TO 33
   32 FF=1.0/(2.0*ALOG(3.7/RR)/2.302585)**2
   33 RETURN
      END
```

SUBROUTINE QSF

```
      SUBROUTINE QSF(H,Y,Z,NDIM)
C     THIS SUBROUTINE DOES THE NUMERICAL INTEGRATION
C     OF A GIVEN FUNCTION USING SIMPSON'S RULE AND
C     NEWTON'S 3/8 RULE
C

      DIMENSION Y(6),Z(5)
      HT=.3333333*H
      IF(NDIM-5)7,8,1

C     IF NDIM IS GREATER THAN 5
C     PREPERATIONS OF INTEGRATION LOOP
C

    1 SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(3)+AUX1+Y(5))
      AUX2=HT*(Y(1)+3.875*(Y(2)+Y(5))+2.625*(Y(3)+Y(4))
     *+Y(6))
      SUM2=Y(5)+Y(5)
      SUM2=SUM2+SUM2
      SUM2=AUX2-HT*(Y(4)+SUM2+Y(6))
      Z(1)=0.
      AUX=Y(3)+Y(3)
      AUX=AUX+AUX
      Z(2)=SUM2-HT*(Y(2)+AUX+Y(4))
      Z(3)=SUM1
      Z(4)=SUM2
      IF(NDIM-6)5,5,2

C     INTEGRATION LOOP
C

    2 DO 4 I=7,NDIM,2
      SUM1=AUX1
      SUM2=AUX2
      AUX1=Y(I-1)+Y(I-1)
      AUX1=AUX1+AUX1
      AUX1=SUM1+HT*(Y(I-2)+AUX1+Y(I))
      Z(I-2)=SUM1
      IF(I-NDIM)3,6,6
    3 AUX2=Y(I)+Y(I)
      AUX2=AUX2+AUX2
      AUX2=SUM2+HT*(Y(I-1)+AUX2+Y(I+1))
```

```
         SUBROUTINE  QSF            ...(CONT'D)

    4 Z(I-1)=SUM2
    5 Z(NDIM-1)=AUX1
      Z(NDIM)=AUX2
      RETURN
    6 Z(NDIM-1)=SUM2
      Z(NDIM)=AUX1
      RETURN
C
C       END OF INTEGRATION LOOP
C
    7 IF(NDIM-3)12,11,8
C
C       NDIM IS EQUAL TO 4 OR 5
C
    8 SUM2=1.125*HT*(Y(1)+Y(2)+Y(2)+Y(2)+Y(3)+Y(3)+Y(3)
     *+Y(4))
      SUM1=Y(2)+Y(2)
      SUM1=SUM1+SUM1
      SUM1=HT*(Y(1)+SUM1+Y(3))
      Z(1)=0.
      AUX1=Y(3)+Y(3)
      AUX1=AUX1+AUX1
      Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
      IF(NDIM-5)10,9,9
    9 AUX1=Y(4)+Y(4)
      AUX1=AUX1+AUX1
      Z(5)=SUM1+HT*(Y(3)+AUX1+Y(5))
   10 Z(3)=SUM1
      Z(4)=SUM2
      RETURN
C
C       NDIM IS EQUAL TO 3
C
   11 SUM1=HT*(1.25*Y(1)+Y(2)+Y(2)-.25*Y(3))
      SUM2=Y(2)+Y(2)
      SUM2=SUM2+SUM2
      Z(3)=HT*(Y(1)+SUM2+Y(3))
      Z(1)=0.
      Z(2)=SUM1
   12 RETURN
      END
```

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